Analyzing Vibration Behavior of Nano FGM (Si_3N_4/SUS304) Plates: Impact of Homogenization Models and Nano Parameters

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This study investigates the response of small-scale length parameters and homogenization models of a simply supported nano-plate composed of functionally graded material. The natural frequency is presented for all cases, and the effect of different modes (Voigt, Reuss, LRVE, and Tamura), thickness ratio, and non-local parameter on the natural frequency is analyzed. The results show that the homogenization scheme is more influential in the vibrational response of FGM nanoplate with lower aspect ratios, and an increase in the small scale parameter causes a decrease in the natural frequency. To derive the governing equations and resolve them, the virtual work principle and Navier's model were employed. The accuracy of the proposed analytical model was verified by comparing the results with those obtained from other models available in the literature.

Keywords: Functionally Graded Material, Small-Scale Length Parameter, Homogenization Models, Natural Frequency, Vibrational Behavior.

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1. INTRODUCTION

In recent years, there has been a significant rise in the utilization of micro/nanostructures in various fields of engineering and technology. The fascinating examples include solar cell energy, micro/nanosensors [1], biological applications, and micro/nanoelectromechanical systems (MEMS/NEMS), among others. However, it is crucial to note that the behaviors of these structures, such as nanoplates, nanobeams, and nanoshells, are vastly different from those of macrostructures due to the influence of small-scale length parameters on their behavior. Therefore, it is essential to develop a profound understanding of the mechanical and thermal behavior of micro/nanostructures.

Among the various types of advanced composite materials used in several sectors, Functionally Graded Materials (FGMs) have drawn significant attention[2-6]. In this regard, the present article investigates the effects of various homogenization models on the free vibration of a functionally graded material (FGM) nano plate. The Voigt, Reuss, Tamura, and Local Representative Volume Elements (LRVE) schemes were utilized to predict the effective material properties of the two-phase particle composite. The results indicate that the Voigt model overestimates frequencies, and the LRVE model provides a good balance between estimation accuracy and ease of implementation.

2. NONLOCAL ELASTICITY ERINGEN THEORY

Eringen proposed that the stress field at a point in an elastic material depends not only on the strain at that point but also on strains at all other points of the body. This nonlocal stress field can be expressed as an integral of the product of the nonlocal modulus and the classical macroscopic stress tensor over the volume of the material.

Eringen showed that the nonlocal constitutive equation can be represented in an equivalent differential form. This is expressed as an equation involving the Laplacian operator acting on the stress tensor [7], with a material constant τ related to the internal and external characteristic lengths.

$$(1-\tau^2L^2\nabla^2)\sigma = t, \tag{2.1}$$

$$\frac{\mu}{L^2} = \left(\frac{e_0\bar{a}}{L}\right)^2, \tag{2.2}$$

where $\mu = (e_0\bar{a})$, e_0 is a material constant and $\bar{a}$ and L are the internal and external characteristic lengths, respectively.

3. PROBLEM DEFINITION GEOMETRY

The problem assumes that the domain has geometry of a nano rectangular plate, depicted in Fig. 1, with a thickness of "h", a length of "a" and a width of "b".

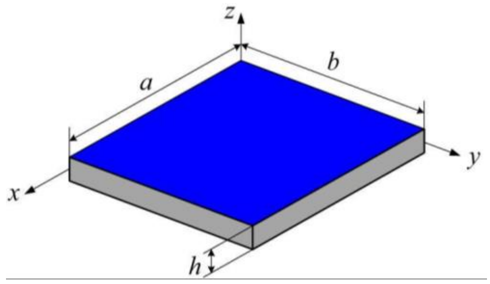

Fig. 1 – The geometry of functionally graded nanoplates

The top surface of the plate consists of a ceramic-rich material (Si_3N_4) while the bottom surface is made of a metal-rich material (SUS304). The material proper-

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ties, including the mass density ρ and Young's modulus E, are determined as follows: $\rho c = 2{,}370$ for (Si_3N_4) and for SUS304. Poisson's ratio$v$ is assumed to be constant at 0.3 for this study.

The volume distribution fraction through the thickness has been identified as following functions.

$$V(z) = \left(\frac{2z+h}{2h}\right)^p. \quad (3.1)$$

4. DIFFERENT MICROMECHANICS MODELS

The micromechanics models chosen for the comparison study are [4]. The Mixture law or Voigt model is a mathematical model used to describe the behavior of composite materials. It combines the properties of individual components linearly to obtain the properties of the composite material. This model helps to calculate various properties of the composite material, such as the effective modulus of elasticity and strength. It was first introduced by Voigt et al in 1889 [8].

$$P(z) = P_c V(z) + P_m(1 - V(z)). \quad (4.1)$$

The Reuss model is a mathematical model used to calculate the effective properties of a composite material. It assumes that the properties of the composite material are obtained by averaging the properties of the individual components. It is used to calculate the effective modulus of elasticity, strength, and other properties of a composite material used to calculate the proprieties of FG structures with the assumption that the stress is uniform through the thickness. Reuss A. (1929) [9].

$$P(z) = \frac{P_c P_m}{P_c(1-V(z)) + P_m V(z)}. \quad (4.2)$$

The Tamura model is a mathematical model used to calculate the effective properties of a composite material. It is based on the concept of strain energy density and assumes that the properties of the composite material are obtained by combining the properties of the individual components in a non-linear manner. It is used to calculate the effective modulus of elasticity, strength, and other properties of a composite material.

The method of Tamura is another way to express the linear law of Voigt where the empirical term q "stress-to-strain transfer" has been added in formulation. Tamura (1973) [10].

$$P(z) = \frac{(1-V(z))\, P_m(q-P_c) + V(z)\, P_c}{P_c(1-V(z)) + P_m V(z)(1-V(z))\,(q-P_c) + V(z)\, P_c(q-P_m)}, \quad (4.3)$$

where $P(z)$ is the effective material property. P_m and P_c are the properties of the Metal and Ceramic faces of beam respectively.

The LRVE is developed based on the assumption that the microstructure of the heterogeneous material is known. The input for the LRVE for the deterministic micromechanical framework is usually volume average or ensemble average of the descriptors of the microstructures.

$$P(z) = P_m[1-, V(z)^{\frac{1}{3}}(1 - \frac{P_m}{P_c}). \quad (4.4)$$

5. GOVERNING EQUATIONS

5.1 The Displacement Field

The displacement field of a theory is determined based on three assumptions: partitioning of in-plane and transverse displacements into bending and shear components, similarity of bending parts of in-plane displacements to classical plate theory, and hyperbolic variation of shear strains causing shear stresses to vanish at the top and bottom surfaces of the plate. The resulting displacement field is then provided.

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x}$$
$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y}, \quad (5.1)$$
$$w(x,y,z,t) = w_b(x,y,t) + w_s(x,y,t)$$

$f(z)$ is the warping function written as:

$$f(z) = z - (1/2 - \frac{h}{z}). \quad (5.2)$$

5.2 The Nonlocal Constitutive Relations

The two-dimensional nonlocal constitutive relations for elastic FG nano-plate can be expressed as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix} - \mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix} = \begin{bmatrix}C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{66} & 0 & 0\\ 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & C_{44}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}, \quad (5.3)$$

where $(\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{yx})$ and $(\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz})$ are the stress and strain components, respectively. The stiffness coefficients, C_{ij}, can be expressed as:

$$C_{11} = C_{22} = \frac{E(z)}{1-v(z)^2}, C_{12} = \frac{vE(z)}{1-v(z)^2}, C_{44} = C_{55} = C_{66} = \frac{E(z)}{2[1+v(z)]}. \quad (5.4)$$

5.3 Hamilton's Principle

Hamilton's principle is used herein to derive the equations of motion.

$$0 = \int_0^t (\delta U - \delta K)dt, \quad (5.5)$$

where δU is the variation of strain energy; and δK is the variation of kinetic energy.

6. SOLUTION PROCEDURE

Following the Navier solution procedure, we assume the following solution form for u_0, v_0, w_b and w_s that satisfies the boundary conditions:

$$\begin{Bmatrix}u_0\\ v_0\\ w_b\\ w_s\end{Bmatrix} = \begin{Bmatrix}U_{mn}e^{i\omega t}cos(\alpha x)sin(\beta y)\\ V_{mn}e^{i\omega t}sin(\alpha x)cos(\beta y)\\ W_{bmn}e^{i\omega t}sin(\alpha x)sin(\beta y)\\ W_{smn}e^{i\omega t}sin(\alpha x)sin(\beta y)\end{Bmatrix}, \quad (6.1)$$

where U_{mn}, V_{mn}, W_{bmn} and W_{smn} are arbitrary parameters to be determined, ω is the eigenfrequency associated with (m, n) th eigenmode. The analytical solutions can be obtained from

$$\left(\begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{12} & a_{22} & a_{23} & a_{24}\\ a_{13} & a_{23} & a_{33} & a_{34}\\ a_{14} & a_{24} & a_{34} & a_{44}\end{bmatrix} - \lambda\omega^2\begin{bmatrix}m_{11} & 0 & 0 & 0\\ 0 & m_{22} & 0 & 0\\ 0 & 0 & m_{33} & m_{34}\\ 0 & 0 & m_{34} & m_{44}\end{bmatrix}\right)\begin{Bmatrix}U_{mn}\\ V_{mn}\\ W_{bmn}\\ W_{smn}\end{Bmatrix} = \begin{Bmatrix}0\\ 0\\ 0\\ 0\end{Bmatrix} \quad (6.2)$$

$$
\begin{aligned}
a_{11} &= A_{11}\alpha^2 + A_{66}\beta^2 \\
a_{12} &= \alpha\beta(A_{12} + A_{66}) \\
a_{13} &= -\alpha[B_{11}\alpha^2 + (B_{12} + 2B_{66})\beta^2] \\
a_{14} &= -\alpha[B_{11}^s\alpha^2 + (B_{12}^s + 2B_{66}^s)\beta^2] \\
a_{22} &= A_{66}\alpha^2 + A_{22}\beta^2 \\
a_{23} &= -\beta[(B_{12} + 2B_{66})\alpha^2 + B_{22}\beta^2] \\
a_{24} &= -\beta[(B_{12}^s + 2B_{66}^s)\alpha^2 + B_{22}^s\beta^2] \\
a_{33} &= D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 \\
a_{34} &= D_{11}^s\alpha^4 + 2(D_{12}^s + 2D_{66}^s)\alpha^2\beta^2 + D_{22}^s\beta^4 \\
a_{44} &= H_{11}^s\alpha^4 + 2(H_{12}^s + 2H_{66}^s)\alpha^2\beta^2 + H_{22}^s\beta^4 + A_{55}^s\alpha^2 + A_{44}^s\beta^2 \\
m_{11} &= m_{22} = I_0 \\
m_{33} &= I_0 + I_2(\alpha^2 + \beta^2) \\
m_{34} &= I_0 + J_2(\alpha^2 + \beta^2) \\
m_{44} &= I_0 + K_2(\alpha^2 + \beta^2) \\
\lambda &= 1 + \mu(\alpha^2 + \beta^2)
\end{aligned}
$$

$(I_0, I_1, J_1, I_2, J_2, K_2)$ are mass inertias defined as:

$$(I_0, I_2, J_2, K_2) = \int_{-h/2}^{h/2} (1, , z^2, zf, f^2)\rho(z)dz$$

And A_{ij}, B_{ij}, D_{ij}, etc., are the plate stiffness, defined by

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}(1, z, f^2(z))\{v\}dz$$

$$(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s) = (A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s)$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} C_{44}[g(z)]^2 dz$$

7. RESULTS AND DISCUSSION

This section of the study focuses on investigating Homogenization Models and small-scale length parameters response of of a simply supported nano-plate composed of functionally graded material. The non-dimensionalized natural frequency is presented for all cases defined as:

$$\bar{\omega} = \omega h \sqrt{\frac{\rho_c}{G_c}}, \qquad (7.1)$$

where $\bar{\omega}$ is the natural frequency, ρ_c and G_c are the mass density and shear modulus of the ceramic phase, respectively.

7.1 Comparison and Validation Numerical Study

The main purpose of Table 1 is to validate the numerical results obtained in this study that investigates the natural frequency of FGM square nano plates with gradient index taken $p = 5$, by comparing them with the results presented in a relevant literature by Natarajan et al. (2012) [11]. The table presents important details regarding the Effect of thickness ratio L/h, modes (1,2 and 3), Homogenization Models (Voigt Reuss, LRVE and Tamura) and the nonlocal parameter μ on the natural frequency. The table allows for a comparison of the results obtained in the current study with the results obtained in the literature, thereby validating the numerical analysis and ensuring the accuracy and reliability of the study.

Table 1 – Effect of thickness ratio, modes, homogenization models, and nonlocal parameter on natural frequency of FGM square nano plates with gradient index $p = 5$

		Mode 1 ($n = 1 - m = 1$)				**Mode 2 ($n = 1 - m = 2$)**				**Mode 3 ($n = 1 - m = 3$)**			
		The nonlocal parameter μ				**The nonlocal parameter μ**				**The nonlocal parameter μ**			
a/h	**schemes**	**0**	**1**	**2**	**4**	**0**	**1**	**2**	**4**	**0**	**1**	**2**	**4**
	Mori-Tanaka [11]	0.0441	0.0403	0.0374	0.0330	0.1051	0.0860	0.0745	0.0609	0.1051	0.0860	0.0746	0.0610
10	Voigt	0.0442	0.0401	0.0376	0.0331	0.1050	0.0863	0.0745	0.0610	0.1980	0.1400	0.1150	0.0888
	Reuss	0.0455	0.0418	0.0385	0.0339	0.1080	0.0884	0.0766	0.0627	0.203	0.144	0.118	0.0914
	LRVE	0.0467	0.0425	0.0397	0.0350	0.113	0.0922	0.0796	0.0653	0.210	0.148	0.121	0.0947
	Tamura	0.0455	0.0418	0.0385	0.0339	0.108	0.0884	0.0766	0.0627	0.202	0.144	0.118	0.0914
	Mori-Tanaka [11]	0.0113	0.0103	0.0096	0.0085	0.0278	0.0228	0.0197	0.0161	0.0279	0.0228	0.0198	0.0162
20	Voigt	0.0113	0.0104	0.0096	0.0084	0.0279	0.0229	0.0198	0.0163	0.0544	0.0388	0.0317	0.0246
	Reuss	0.0118	0.0107	0.0099	0.0087	0.0288	0.0237	0.0203	0.0167	0.0565	0.0399	0.0326	0.0254
	LRVE	0.0120	0.0110	0.0102	0.0089	0.0298	0.0244	0.0212	0.0173	0.0582	0.0412	0.0338	0.0260
	Tamura	0.0117	0.0107	0.0099	0.0087	0.0288	0.0237	0.0205	0.0167	0.0565	0.0399	0.0326	0.0254

7.2 The Effect of Different Modes on the Natural Frequency

According to Fig. 2, there is a clear relationship between the height values of natural frequency and their corresponding eigenmode values. This means that as the values of natural frequency increase, while eigenmode values also increase. Additionally, the smallest natural frequency values are associated with the smallest eigenmode values.

Overall, Fig. 2 shows how different eigenmodes affect the natural frequency of an FGM nano plate, and emphasizes the significance of selecting an appropriate analytical solution to accurately determine the natural frequency.

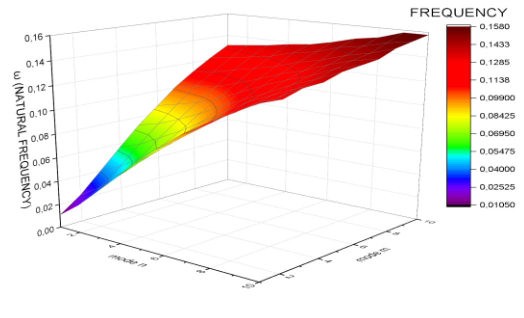

Fig. 2 – The effect of different modes on the natural frequency of FGM nanoplate (3D) plot

7.3 Effect of the Thickness Ratio Parameter (*a*/*h*) on the Natural Frequency

Fig. 3 presents the impact of homogenization schemes on the fundamental frequency of FGM nano-plate at different aspect ratios (*a*/*h*), where $p = 5$. The results show that the homogenization scheme is more influential in the vibrational response of FGM nano-plate with lower aspect ratios, and this conclusion applies to all types of models.

Moreover, it is observed that the Voigt and Reuss modelshave the highest and lowest frequencies, respectively, among all homogenization schemes. On the other hand, increasing the number of material length scale parameters leads to an increase in the fundamental frequency, indicating that the LRVE model has the highest frequency among other plate models.

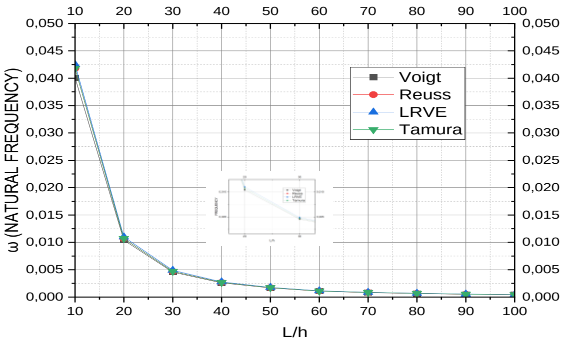

Fig. 3 – Effect of the nonlocal parameter (μ) and the thickness parameter (*a*/*h*) on natural frequency

7.4 Effect of the Nonlocal Parameter (μ) and the Homogenization Models on the Natural Frequency

Fig. 4 illustrates the correlation between the nonlocal parameter and the natural frequency of (FGM) nano-plate under different homogenization models.

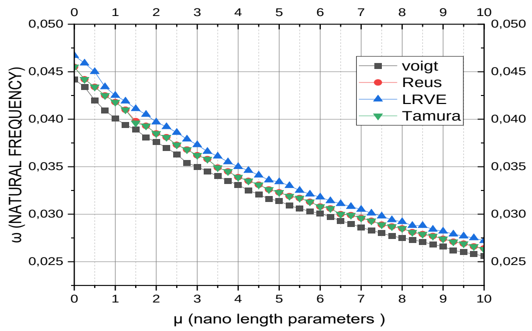

Fig. 4 – Effect of the nonlocal parameter (μ) and thehomogenization scheme on natural frequency

The natural frequency of the plate diminishes as the nonlocal parameter increases, resulting in a stiffer structure. Of the models employed, the Voigt model shows the least significant frequency decrease, while the LRVE model exhibits the most prominent. The Tamura and Reuss models also demonstrate a frequency reduction, albeit less pronounced than the Voigt and LRVE models. The frequency decline is attributed to the amplified stiffness of the nano plate owing to the nonlocal parameter.

7.5 Effect of the Nonlocal Parameter (μ) and Eigenmode on Natural Frequency

Fig. 5 displays how the natural frequencies of an FG nano-plate with $a/h = 10$ are impacted by the first three eigenmode values for different small scale parameter values. The results show that an increase in the small scale parameter causes a decrease in the natural frequency. This can be attributed to the reduction in stiffness of the FG nano-plate caused by an increase in the small scale parameter.

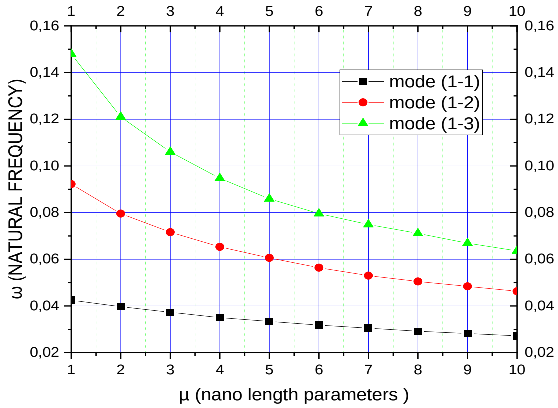

Fig. 5 – Effect of the nonlocal parameter (μ) and modes on natural frequency

8. CONCLUSION

The study provides valuable insights into the vibrational behavior of functionally graded material nano-plates under different conditions. The results show that the homogenization scheme, aspect ratio, and small-scale length parameter significantly affect the natural frequency of the nano-plate. The study confirms the accuracy and reliability of the numerical study and provides important resources for researchers and engineers working in advanced composites sectors. The findings can be used to design and optimize nano-plates in various applications.

REFERENCES

1. B. Jain, G. Sancheti, V. Jain, *J. Nano- Electron. Phys.* **13** No 2, 02006 (2021)
2. B. Rebai, A. Bouhadra, A.A. Bousahla, *Archive of Applied Mechanics* **91** No 7, 3403 (2021).
3. M. Chitour, A. Bouhadra, M. Benguediab, K. Mansouri, A. Menasria, A. Tounsi, *J. Nano- Electron. Phys.* **14** No 3, 03028 (2022).
4. B. Rebai, K. Mansouri, M. Chitour, A. Berkia, T. Messas, F. Khadraoui, B. Litouche, *J. Nano- Electron. Phys.* **15** No 1, 01022 (2023).
5. B. Rebai, *AIMS Materials Science Journal* **10** No 1, 26 (2023).
6. A. Berkia, M. Benguediab, A. Bouhadra, K. Mansouri, A. Tounsi, M. Chitour, *J. Nano- Electron. Phys.* **14** No 3, 03031 (2022).

7. A.C. Eringen, *J. Appl. Phys.* **54**, 4703 (1983).
8. M.M. Gasik, *Comput. Mater. Sci.* **13** No 1-3, 42 (1998).
9. J.R. Zuiker, *Comput. Eng.* **5** No 7, 807 (1995).
10. T. Itoh, T. Tamura, T. Matsumoto, *J. Am. Oil Chem. Soc.* **50**, 122 (1973)
11. S. Natarajan, S. Chakraborty, M. Thangavel, S. Bordas, T. Rabczuk, *Comput. Mater. Sci.* **65**, 74 (2012).

Аналіз вібраційної поведінки пластин Nano FGM (Si_3N_4/SUS304): вплив моделей гомогенізації та нанопараметрів

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У цьому дослідженні досліджується реакція дрібномасштабних параметрів довжини та моделей гомогенізації просто підтримуваної нанопластини, що складається з функціонально сортованого матеріалу. Власна частота представлена для всіх випадків, а також проаналізовано вплив різних режимів (Войгта, Рейсса, LRVE і Тамури), коефіцієнта товщини та нелокального параметра на власну частоту. Результати показують, що схема гомогенізації має більший вплив на вібраційну реакцію нанопластини FGM із меншими співвідношеннями сторін, а збільшення параметра малого масштабу спричиняє зменшення власної частоти. Щоб вивести керівні рівняння та розв'язати їх, було використано принцип віртуальної роботи та модель Нав'є. Точність запропонованої аналітичної моделі було перевірено шляхом порівняння результатів з результатами, отриманими з інших моделей, доступних у літературі.

Ключові слова: Функціонально градуйований матеріал, Дрібномасштабний параметр довжини, Моделі гомогенізації, Власна частота, Вібраційна поведінка.