A NON-PARAMETRIC APPROACH TO DETERMINE AN EFFICIENT PREMIUM FOR DROUGHT INSURANCE

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Abstract: *Insurance to deal with prolonged drought periods in rural Africa requires a practical method to estimate accurate premium values that minimize economic losses. We use non-parametric methods to determine the risk non-neutral insurer's premium for drought insurance on rain-fed crops. Premium values are estimated on the basis of percentage of the expected yield losses over the potential yields. Expected yield losses are estimated based on data on the levels of rainfall, potential evapotranspiration and water-holding capacity of the soil, and water requirement of the crop. Maize crop in West Kenya, and rice crop in the Central High Plains of Madagascar are taken as case studies. To check if farmer's choice of starting seasons affects the expected yields and the values of premium, we employ forecasted yields for two different sowing dates (October vs. November) for maize, and two different transplantation dates (November vs. December) for rice. The mean-variance (E-V), the First-Degree Stochastic Dominance (FSD), and the Second-Degree Stochastic Dominance (SSD) efficiency criteria are used to rank each pair of distributions. Results show that an insurer for maize production in Western Kenya would require a premium value between 43 and 55% of the potential yields to fully cover the loss caused by lack of rainfall. Under E-V and FSD, the two yield distributions cannot be ranked, but under SSD the yield distribution of the October-sown maize dominates that of November. For lowland rice in the Central High Plains of Madagascar, all three efficiency criteria indicate that the yield distribution of the December-transplanted rice dominates that of November and the premium values are less than 4 % of the potential yields.*

Keywords: drought insurance, non-parametric methods, stochastic dominance, Africa.

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1. Introduction

Risk and uncertainty are inseparable to crop production. With climate change causing extreme weather conditions, the need to insure against production risks to boost agricultural revenue and improve food security is growing. Many African countries suffering from chronic food insecurity while experiencing extreme drought are in need of drought insurance to limit the negative impacts of insufficient rainfall on farm income and food availability. However, in rural Africa, because of incomplete and asymmetric information, setting a drought insurance with accurate and undistorted premium values that allow both contracting agents to benefit from the insurance has been difficult.

The objective of this paper is to determine drought insurance premium based on the efficient uses of forecasted yield index. The method involves a risk non-neutral insurer, and the premium values are estimated on the basis of the statistical distribution of the percentage of expected yield losses over the potential yields. Expected yields are estimated using data on the levels of rainfall, potential evapotranspiration and water-holding capacity of the soil, and water requirement of the crop. Two main food crops from the largest production areas in two African countries are used as case studies: maize in Western Kenya, and lowland rice in the Central High Plains of Madagascar. The results provide policy implications for premium determination in drought insurance for public and private insurers operating in rural areas.

2. Literature review

Crop Water Needs

Water and irrigation play significant roles for crop growth. Lack of water affects plant's growth due to stress, while excess humidity causes soil nutrient leaching and induces crop disease (Baldy and Stigter, 1997). As a result, both insufficient and excessive amounts of rainfall reduce crop yields and harvest. The amount, intensity, frequency, and timing of rainfall and irrigation, as well as farm practices are important in determining the effectiveness of any means of satisfying crop water needs (Mapp and Eidman, 1976; Hornbaker and Mapp, 1988; Talpaz and Mjelde, 1988; FAO, 1991, 1996; Admire, Ray and Hefner, 1997).

Specifically, Hornbaker and Mapp (1988), employed simulation and recursive programming methods to conclude that timing of irrigation was important for sorghum crops for a profit maximizing farm. They monitored irrigation on a daily basis and used weather and soil data. Similarly, Talpaz and Mjelde (1988) proposed an optimizing simulation method for scheduling crop irrigation and particularly corn irrigation in the High Plain area of Texas. They examined the importance of the frequency and quantity of water used for plants while varying the soil moisture. Similarly, Admire, Ray and Hefner (1997) found that practices like the use of "side inlet irrigation system" field appeared to be more beneficial in reducing water management cost than the usual "cascade flood irrigation" system for irrigated rice production.

Crop and rainfall insurances

As the lack or excess of rainfall causes huge production risks, crop insurance has become highly necessary. However, establishing a crop insurance in developing countries has been difficult. Early studies (e.g. Hazell, Pomerada and Valdes.1986; and Roberts and Dick, 1991) concluded that developing countries' experiences on crop insurance showed mixed results. Ray (1974) and Quiggin (1991) concluded that the main difficulties of implementing crop insurance in developing countries stemmed from the lack of reliable data on crop and yields, wide variety of agricultural practices, and inadequate market infrastructure. Additionally, the high monitoring cost to minimize moral hazard and adverse selection (Arnott and Stiglitz, 1991), and financial constraints have limited the use of crop insurance.

Rainfall insurance is among crop insurances that have been employed to deal with crop production risks due to inadequate amount of rainfall in developing countries (Mishra, 1995). Rainfall insurance like any other crop insurance has two general objectives at the farm level: to reduce the variability of farm income, and to ensure some subsistence level of living and debt repayments, particularly after a devastating drought or flooding. particularly attractive to farmers and insurers.

Premium determination for crop insurance

Hazell (1999) and Skees (2000) outlined the main advantages of rainfall insurance for developing countries. The ease of monitoring and getting accessible information on rainfall measurement reduces moral hazard and adverse selection considerably and makes the use of rainfall insurance attractive (Hazell, 1992; Hazell, 1999). But the complex computation of the premium, requiring precise data on soil and crop characteristics especially in the developing country context, has been the insurance's main stumbling block. Several hints from crop insurance can shed lights into the difficult practices for setting premium. For instance, Ray (1974) focused on insuring food grains in selected developed (such as the US and Japan) and developing countries (such as India and Sri-Lanka). He proposed the determination of insurance coverage and premium rates based on a two-stage actuarial method.

Similarly, Hazell, Bassoco, and Arcia (1986), studying the Mexican crop insurance model shed more light into the practical methods of determining the premium. They presented three different schemes of premium setting and assumed that total premium would be equal to the total indemnity. The first scheme calculates premium based on indemnity equal to the proportion of yields shortfall (relative to the normal yields) times the amount of credit borrowed for the crop and augmented by the amount that had to be paid to the bank as administrative cost. Thus, indemnities were considered as a contribution to credit payments.

The second scheme expresses premium as the simple difference between the actual and expected revenue. Those two premium designs refer to Gerber (1979)'s denomination as "loss principles". The third scheme is a combination of the two principles. Again, the difficulty, especially with the first scheme is the ambiguity in setting the augmented amount to be paid to the bank. Yet, another problem is the assumption that premium should be exactly the same amount as the indemnity in each case; it ignores the attitude of the insurer towards risk.

A premium design like that for sugar cane insurance in Mauritius in the 70's involves assigning a rank to each farmer, and setting a premium schedule based on the rank between two periods (Ray 1974). The rank is inversely related to the ratio of total indemnity over total premium from last period. The lower the rank in the present period, the larger is the premium rate to be paid in the next period. Moral hazard in this case is limited, because each year farmers are making efforts to reduce the ratio indemnity/total premium, and to improve their ranking in the next period.

Moreover, compensation in the example of Mauritius sugar cane is calculated as the value of the total insurance coverage deducted by the first loss and by the actual harvest times the proportion of yield shortfalls and valued at the current price. And for most of the reported studies (e.g. Schoney, Taylor and Hayward, 1994; Schwanz 1997) calculation of the indemnity payments relies on the direct measure of the yield shortfalls, which subsequently distorts payment allocation due to moral hazard and adverse selection. Even in developed countries, monitoring directly the yields to determine premium and indemnity presents enormous difficulty, and is often costly (Gardner, 1994).

Premium for rainfall and drought insurance

For rainfall or drought insurance particularly, Skees, Black and Barnett. (1997) gave some insight into how rainfall index insurance might work and how premium and indemnity can be determined.1 They proposed three different methods of calculation. The first method was called the "zero-one" contract, based on rainfall distribution and identified the premium rate as the probability that the level of rainfall would fall below certain value or strike level, at which the farmer wants to be insured.

The second method was called a "layered contract" which was derived from the first method but with different brackets of level of rainfall. The third was called a "percentage contract", obtained by developing payouts as a function of a rainfall amount below the strike level. In the latter method, the premium was expressed as some percentages to the total indemnity and obtained by taking the percentage of the difference between the actual amount of rainfall and the strike level over the strike level multiplied by the dollar amount of liability.

[1] See also Martin, Barnett and Coble (2001).

More recently, Muna, Purnaba and Setiawaty (2019) devised a rule for premium determination based on rainfall index for rice in Bongor, Indonesia. They designed a 'trigger' and an 'exit' to insurance benefit based on rainfall data. The trigger is the benchmark value of rainfall below which partial risk of crop failure may occur assigning the right to the policyholder to claim some partial benefit payment. When rainfall level is above the trigger level, farmers cannot claim any indemnity from the insurer. The exit level is the critical value of rainfall below which total crop failure occurs and gives the right to the policyholder to submit full benefit claim. In their model, the premium wss calculated based on the expected values of the claim (adding some administrative costs) using Bayesian probability approach and the distribution of the levels of rainfall for several years.

It is worth noting that beside the assumption of zero profit or perfect competition in the insurance market, many of the past studies on either premium or indemnity setting did not put much emphasis on the role and attitude towards risk of the insurer. Insurers were often assumed to be part of a public institution and considered as risk-neutral. In the modeling of risk management tools such as price stabilization and buffer stock, the assumption that the principal agent, the government, is risk neutral has been customary (Newbery, and Stiglitz, 1989; Herman, 1993). But with the growing number of insurers with various asset portfolios and attitudes towards risks, such an assumption is no longer tenable. Therefore, our model particularly assumes that insurers have some degrees of risk aversion.

3. Theory

In general, any premium calculation is a rule H(.), relating the indemnity to the farmers or payment *S*, a random variable, to the premium value *p*, which is a real number:

$$p = H(S)\,. \tag{1}$$

Therefore the distribution of *S* and the mathematical expression of *H*(.) have to be determined in order to obtain the premium p.

The indemnity or payment *S* is directly related to another random variable *B* that affects the harvest or the yields. As our sole focus is on the impact of rainfall deficit, we temporarily assume that excess water can be managed to cause no damage to crop yields. In that regard, *B* represents the water balance or humidity stored in the soil. The value of *B* depends mainly on the amount of rainfall, the water holding capacity of the soil, and the water requirement of the crop.

The payment *S* is expressed as

$$\text{S} = \text{f (B)}, \tag{2}$$

where $f' < 0$, because payment or indemnity to the farmers decreases as more water from rainfall is stored in the soil for the benefit of the crop's growth.

To determine the expression of the rule *H* (.) in equation (1), two fundamental sets of assumptions are being made: (i) the insurer is not indifferent toward risk and has a constant coefficient of risk aversion with positive marginal utility of income, and (ii) the premium is set fairly. Fairness is expressed under the principle of *zero utility*, as it is defined by Gerber (1979), for which the expected utility of return for the insurer is equal to the utility from the insurer's initial wealth. This is a plausible assumption especially when the insurance market is highly competitive. These two assumptions can be expressed as follows:

$$U(x) = (1 - e^{-ax})/a \tag{3}$$

$$U(x_0) = E[U(x_0 + p - S)] \tag{4}$$

where *U*(.) indicates the insurer's utility function, *x* is the return, *a* is the coefficient of risk aversion, x_0 represents initial wealth, E is the expectation operator, and the quantity *p* - *S* (or premium charged minus payment) represents the random return of the insurer. Setting the initial return arbitrarily to $x_0 = 0$ and solving the system of equation (3) and (4) for *p*, we show in Appendix 1 that the premium can be expressed as:

$$p = \frac{1}{a}\ln(E[e^{aS}]) \tag{5}$$

The parameter *a* can be given different values in the reasonable range noting that the premium increases as the insurer's risk aversion increases, as it is implied by the equation (5). Alternatively, after making the substitution of (2) into (5), the latter can be rewritten as $p = \frac{1}{a}\ln(E[e^{af(B)}])$. Also, it is important to note that in equation (5), the term inside the logarithm function is just the moment generation function of the distribution of *S* for smaller values of coefficient of risk aversion a.

4. Method

The Food and Agriculture Organization of the United Nations (FAO) has been using an index that represents the extent of yield loss due to water stress over the entire crop's growth period, i.e. from the sowing and harvest times. The index measurement relies upon available information regarding soil and crop characteristics, crop growth stage, and the amount of rainfall (Dastane, 1974; FAO, 1991, 1996). The method divides the growing period into several intervals of time (say, week or month). The calculation follows a time discrete dynamic process and comprises of two steps.

The first step consists of estimating the water balance in the soil and attaching a corresponding index to it at the end of each time period over the crop growing. The water balance in the soil at a point in time t, denominated B_t, and available to the plant for the next time interval period between *t* and *t+1* is calculated as the water balance requirement satisfaction in the current period t and the water retained in the soil from the last period:

$$B_t = (PR_{at} - K_t * PE_t) + B_{t-1}, \tag{6}$$

where PR_{at} is the actual precipitation between *t-1* and *t* ; K_t is called the "crop coefficient"[2] which quantifies the crop's water use at a period *t* and is both crop and growth-stage specific; PE_t[3] is the potential evapotranspiration of the soil; and the variable B_{t-1} is the amount of water retained in the soil at time *t-1*. In equation (6), the quantity (K_t*PE_t) represents the water requirement of the plant between time *t-1* and *t*, which may or may not have been satisfied.

In general, the soil's water supply that affects crop yields for the next period depends on the sign and values of B_t and the maximum water holding capacity of the soil, say, *Rmax* (a positive number). Three different cases are to be considered to assign the FAO index, called ΔI_t in percentile, representing the extent of water stress for the crop between time *t-1* and *t*.

Case 1: $B_t > Rmax$. It indicates that the water balance in the soil exceeds the maximum water holding capacity of the soil. In this case, the requirement of the crop between *t-1* and t was satisfied; additionally, there is an excess water (run-off) that corresponds to the quantity (B_t-*Rmax*). which will be available to the crop for the next period. As it is assumed earlier, excess water can be managed (diverted or there is no crop damage from reasonable amount of excess water; so conventionally ΔI_t =0 in this case.

Case 2: $0<B_t<Rmax$. It means that the crop water requirement between *t-1* and *t* was satisfied and a B_t amount of water is retained in the soil and will be used in the next period (plus, eventually, some future rainfall.) So, as it is in the first case, there was no water stress during *t-1* and *t*, and, again ΔI_t =0.

Case 3: $B_t<0$. It indicates that crop water requirement was not satisfied for the last time period and that would affect crop yield. The water shortage is equivalent to the absolute value of B_t. In this case, the FAO methods suggest the following:

$$\Delta I_t = |B_t| \text{ x } 100/\ \Sigma_t(K_t PE_t). \tag{7}$$

[2] K_t called the "crop coefficient" is defined as the ratio between maximum crop evapotranspiration and a reference crop potential evapotranspiration.
[3]PE_t is the maximum quantity of water evaporated and transpired by a uniform cover of short dense grass when the water supplied is not limited.

Equation (7) defines the extent of water stress as the percentage of the water deficit over the sum of the water requirement of the crop in the entire crop growth period. The second step consists of summing up all the ΔI_t 's for the entire growth period, so that the final index representing the extent of water stress on the crop is:

$$I_f = \Sigma_t \, \Delta It \quad (\%) \tag{8}$$

This index I_f is directly related to the payment variable S (in equation (2)), which is the payment to be made by the insurer (i.e. the benefit to be claimed by the farmer) due to the harvest loss caused by the drought. As it is implied by the definition of the index, the higher I_f, the higher the yield loss. The actual yield loss due to drought is, ceteris paribus, the index I_f multiplied by the potential yield. In other words, the expected yield in each season is 1- I_f times the potential yield. The potential yield is the ideal yield obtained when there was no water stress. For clarity, we express the expected yield and premium in percentage of the potential yield.

5. Procedures and Data

The values of the crop coefficient K_t for each growth stage are obtained from the FAO (1991, 1096) studies that reported the corresponding crop coefficient per growing period in Kenya and Madagascar (Table 1). n Kenya, maize crop is generally planted in two seasons like many other grains. The main season is the so-called winter maize sown in March-April and harvested in October-November. The second maize season is generally sown between October and November, and harvested in March-April.

Table 1. Crop Coefficients Representing Water Needs for Maize and Rice

	Month after sowing (maize) or transplantation (rice)			
	First Month	Second Month	Third Month	Fourth Month
Maize coefficient	0.6	1.1	1.1	0.67
Rice coefficient	1.17	1.37	1.4	1.27

Source: FAO (1991, 1996).

Our empirical study focuses on Western Kenya's second maize season during which drought is more frequent. For that second season of maize production, the most critical periods for the plant in terms of water needs are December and January, which are the second month and third month following the sowing dates respectively. Those critical periods correspond respectively to the pollination and grainfilling stages.

Contrastingly, the lowland rice cycle in the high plains of Madagascar lasts generally 5 months; rice grains are sown on October-November and transplanted between November and January. Rice harvest usually takes place on April-May. The water requirement after transplantation is high especially during the second and third months.

Our calculations of the water holding capacity of the soil use data and soil classifications from the FAO (1991, 1996). Values differ by locations but, for instance, the average water-holding capacities (measured as the equivalent level of precipitation) of the soils for maize crop near Eldoret and for rice near Antananarivo were about 60 mm and 100 mm. Table 2 and Table 3 summarize examples of historical data between October and April and between the crop years 2006-07 and 2015-16 on average rainfall, evapotranspiration, and monthly rainfall for Eldoret (Western Kenya) and Antananarivo (Central High Plains of Madagascar).

Table 2. Potential Evapotranspiration and Precipitation Near Eldoret (in West Kenya)

Month	Potential	Normal Average			Yearly Actual Precipitation							
	Evapotranspiration *	Precipitation *										
	(mm)	(mm)			(mm)							
			2015-16	**2014-15**	**2013-14**	**2012-13**	**2011-12**	**2010-11**	**2009-10**	**2008-09**	**2007-08**	**2006-07**
October	109	41	85	14	140	128	41	21	59	59	39	9
Novembe r	112	54	70	100	40	39	54	620	41	130	130	10
December	112	30	23	50	160	12	160	30	30	3	3	45
January	122	27	27	0	0	63	30	38	45	15	46	61

Table 2 (cont.). Potential Evapotranspiration and Precipitation Near Eldoret (in West Kenya)

Month	Potential	Normal Average			Yearly Actual Precipitation							
	Evapotranspiration *	Precipitation *										
	(mm)	(mm)			(mm)							
February	132	30	55	36	5.8	30	30	30	66	46	9	67
March	132	35	180	850	96	68	13	129	156	121	36	72
April	112	n.a	n.a	193	188.5	24	178	n.a	128	159	192	118
May	97	113	70	54	10	111	87	490	122	152	41	273

Sources: FAO and US Department of Commerce (various years), The Weather Bureau (various years)

*: Average figures are obtained by a fifty-year average (1960-2010) of evapotranspiration and rainfall at each period

Table 3. Potential Evapotranspiration and Precipitation Near Antananarivo (in Central High Plains of Madagascar)

Month	Potential	Normal Average			Yearly Actual Precipitation				
	Evapotranspiration	Precipitation							
	(mm)	(mm)			(mm)				
			2015-16	**2014-15**	**2010-11**	**2009-10**	**2008-09**	**2007-08**	**2006-07**
October	107	49	49	128	49	1	63	44	311
November	109	179	54	17	187	186	115	75	118
December	102	318	395	201	176	389	305	68	246
January	102	254	731	680	81	189	191	363	500
February	104	211	170	211	244	71	287	232	281
March	89	220	543	203	247	48	66	109	314
April	84	60	2	107	79	82	40	58	85
May	69	18	3	13	15	5	159	9	0

Sources: FAO and US Department of Commerce (various years), The Weather Bureau (various years).

*: Average figures are obtained by fifty-year averages (1960-2010) of potential evapotranspiration and rainfall at each period.

Each year, the percentage of the expected or forecasted yields loss (i.e. *S*) due to insufficient rainfall is based on the estimated yield loss index I_t derived earlier in the first step. Furthermore, it is assumed that appearance of each yearly forecasted yield is equally likely. Values of premium were calculated using equation (5) and also expressed in terms of percentage over potential yields. Because of uncertainties on planting season, estimation of the forecasted yields based on rainfall situation were conducted for two different dates of starting season: October vs. November for maize in Eldoret, West Kenya, and November vs. December for lowland rice in the Central High Plains region of Madagascar.

The distributions of forecasted yields were compared using non-parametric statistical method, and especially First-Degree Stochastic Dominance (FSD) and the Second-Degree Stochastic Dominance (SSD) (Hanoch and Levy, 1969; Hadar and Russel, 1969, 1971; Kuosmanen, 2004; Chakrabarty and Swamy, 2014) ordering while maintaining the assumption that the appearances of the forecast values are equally likely during the observation periods.

5. Results

The distributions of the forecasted yields in terms of percentage over the potential yields for the two different starting seasons for maize and rice are presented respectively in Table 4 and Table 5. Overall, the impact of the lack of rainfall on forecasted maize yields in Eldoret (Western Kenya) is highly significant, representing a loss of approximately half of the potential yields if irrigation of the maize crops relies only on rainfall. Contrastingly, on average, the loss is only limited, between 4 and 9% of the potential yields, for the lowland rice near Antananarivo (Central High Plains of Madagascar). A fairly good amount and an adequate distribution of rainfall in combination with high water retention capacity of the rice field are probably the main reasons. However, for both crops, a comparison of the means and variances of yields between the two different seasons reveals that because of rainfall distribution, sowing dates for maize and transplantation dates for rice appear to affect yields significantly.

Table 4. Distribution of the Percentage of Rainfall-Based Forecasted Yields over the Potential Yields of Maize for Two Different Sowing Time Near Eldoret (in West-Kenya): A Non-Parametric Comparison

Crop Calendar Year	October-Sown	Rank	November-Sown	Rank	Absolute Deviations	Rank of the Absolute	Absolute Deviations	Rank of the Absolute
	Maize Yield (%)		Maize Yield (%)		Oct.- Sown Yield	Deviation	Nov.- Sown Yield	Deviation
	X_i	$R(X_i)$	Y_i	$R(Y_i)$	$\|X_i\text{-}Xhat_i\|$	$R(\|X_i\text{-}Xhat_i\|)$	$\|Y_i\text{-}Yhat_i\|$	$R(\|Y_i\text{-}Yhat_i\|)$
15-16	52.2	12	42.4	4	4.2	9	4.9	11
14-15	41.7	3	45.1	9	14.7	16	2.2	3
13-14	82.8	20	49.9	11	26.4	20	2.6	5
12-13	60.9	16	34.9	2	4.5	10	12.4	14
11-12	72.5	19	66.4	17	16.1	17	19.1	18
10-11	69.2	18	54.5	14	12.8	15	7.2	12
09-10	44.5	7	44.1	5	11.9	13	3.2	7
08-09	52.7	13	46.3	10	3.7	8	1	2
07-08	55.5	15	44.9	8	0.9	1	2.4	4
06-07	31.8	1	44.4	6	24.6	19	2.9	6
Sum	T^a =	124		86				
Average	56.4		47.3		T^b =	1946.000		
Standard Deviation	0.154		0.0838					
Source: Author'' s calculations. a: T^a is the sum of rank, used as test statistics; Ho: $E(X)=E(Y)$ vs. Ha: $E(X)>E(Y)$; p-value=0.08.								
b: T^b is the sum of the squared rank, used as test statistics; Ho: $var(X)=varE(Y)$ vs. varE$(X)>varE(Y)$; p-value=0.038.								

Table 5. Distribution of the Percentage of Rainfall-Based Forecasted Yields over the Potential Yields of Lowland Rice for Two Different Transplantation Time Near Antananarivo (in the Central High-Plains of Madagascar): A Non-Parametric Comparison

Crop Calendar Year	Nov. Transplanted	Rank	Dec. Transplanted	Rank	Absolute Deviations	Rank of the Absolute	Absolute Deviations	Rank of the Absolute
	Rice Yield (%)		Rice Yield (%)		Nov. Transplanted Rice	Deviation	Dec. Transplanted Rice	Deviation
	X_i	$R(X_i)$	Y_i	$R(Y_i)$	$X_i\text{-}Xhat_i$	$R(\|X_i\text{-}Xhat_i\|)$	$Y_i\text{-}Yhat_i$	$R(\|Y_i\text{-}Yhat_i\|)$
15-16	86.4	3	100	11.5	4.9	6	3.2	3.5
14-15	79.5	2	100	11.5	11.8	13	3.2	3.5
10-11	100	11.5	99.7	8	8.7	10.5	2.9	1
09-10	100	11.5	90.2	5	8.7	10.5	6.6	8
08-09	97.6	6	100	11.5	6.3	7	3.2	3.5
07-08	77.1	1	87.4	4	14.2	14	9.4	12
06-07	98.2	7	100	11.5	6.9	9	3.2	3.5
Sum	T^a=	42		63				
Average	91.3		96.8		T^b =	751.500		
Standard Deviation	0.098		0.055					
Source: Author's calculations. a: T^a is the sum of rank, used as test statistics; Ho: $E(X)=E(Y)$ vs. Ha:$E(X)>E(Y)$; p-value>>0.1.								
b: T^b is the sum of the squared rank, used as test statistics; Ho: $var(X)=varE(Y)$ vs. $varE(X)>varE(Y)$; p-value=0.021.								

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Non Parametric Testing

Non parametric statistical tests are used to allow comparison between each pair of distributions of forecasted yields over two different periods. The advantage of such a method is that it does not require restrictive conditions such as normality (and no need to test for it either) in comparing the two distributions. Assuming that the two samples are mutually independent and are random samples from their respective population, the *Mann-Whitney* method is used to see if one of the means is significantly greater than the other (*Ho*: $E(x)=E(y)$ vs. *Ha*: $E(x)>E(y)$).

Basically, the *Mann-Whitney* test pools together the forecasted yields for the two different seasons, ranks them from smallest to highest value, and uses the sum of the ranks in either one of the distributions as a test statistic when there are no or only few ties in the ranks. The value of the test statistic is compared to the values on the *Mann-Whitney* Quantiles table in order to obtain the *p*-value.[4]

Similarly, a "squared rank-test for variance" method is used to see whether the two variances from the two different seasons differ (*Ho: var(x)=var(y) vs. Ha: var(x)>var(y).*). The "squared rank-test for variance" looks at the absolute deviations of the yearly forecasted yields from the average for each season, rank them from the smallest to largest values and uses the sum of the squared rank for either one of the two distributions as a test statistics. A table for the quantile of the "squared ranks test statistics" is used to find the rejection region.

First for maize production in Eldoret (Western Kenya), Table 4 indicates that maize sown in October has higher expected forecasted yields but has higher standard deviation than maize sown in November. The *Mann-Whitney* test shows that at a significance level α=0.1 the October-sown maize has indeed higher expected yield, however, at α=0.05, the null hypothesis of equality of means is still maintained. The non-parametric test of equality of variance shows that the null hypothesis is rejected at α=0.05, meaning that yields of the November-sown maize are less variable than of the October -sown.

Second, for rice grown near Antananarivo in the Central High Plains of Madagascar, Table 5 indicates that the December-transplanted rice produces higher yield mean and lower yield variance than the November-transplanted rice. The *Mann-Whitney* test concludes that the null hypothesis cannot be rejected at any reasonable significance level (*p*-value far greater than 0.1), meaning that, statistically, there is no significant difference between the means of the two distributions. However, the "squared variance ranks" test that at α=0.05, the December-transplanted rice yields are less variable than those from rice transplanted in November.

Values of the Premium under Different Assumptions

If the risk averse insurer assumes that farmers are risk averse too, and if information on rainfall-based forecast yield is available to both parties, then the decision about premium calculation of the drought insurance will depend on the choice of the starting season (sowing for maize, and transplantation for rice) which generate two distributions of yields for each crop. The results so far reveal the key parameters of and the compare the yield distributions but remain inconclusive, especially for maize crop, in determining on which crop season shall the premium be determined. Therefore, we use three different criteria to rank the two different seasons for each crop. The first is the mean-variance (*E-V*), criterion which assumes that farmers are risk averse and have approximately a quadratic utility function). The *E-V* criterion indicates that a distribution with higher mean and lower variance will be preferred to a distribution with lower mean and higher variance (Hadar and Russel, 1969; Levy and Markowitz, 1979).

The two other criteria are the First-Degree Stochastic Dominance (FSD) and the Second-Degree Stochastic Dominance (FSD) (Hanoch and Levy 1969; Hadar and Russel, 1969, 1971; Kuosmanen, 2004; Chakrabarty and Swamy 2014). The FSD assumes positive marginal utility of money and states that for a random variable, say, yield Y, a distribution with a cumulative probability density function (cdf) $F(Y)$ first-degree stochastically dominates another distribution with cdf $G(Y)$ if and only if $F(Y) \leq G(Y)$. It implies that the cdf curve $F(Y)$ must always lie below the cdf $G(Y)$. On the other hand, SSD assumes that a distribution with cdf $F(Y)$ second-degree stochastically dominates another distribution with a cdf $G(Y)$ if

[4] See Connover (1980).

$$\int_{-\infty}^{c} F(Y).dY \leq \int_{-\infty}^{c} G(Y).dY,$$

i.e. if within the range of the observed yield values, the area below the cdf $F(Y)$ is smaller than the area below the cdf $G(Y)$. Under both FSD and SSD, the expected value of the yield Y of the stochastically dominant distribution is always higher than that of the non-dominant distribution.

Results of the implications for premium calculation using the non parametric and stochastic dominance ordering methods are summarized in Table 6.

Table 6. Estimated Premium Values for Drought Insurance for Maize and Rice

Methods of Ranking		Maize Crop			Rice Crop		
Efficiency Criteria	Assumptions	Efficient Set	Values of the Premium * (in % of potential yield) a = 0.01	a = 0.0001	Efficient Set	Values of the Premium * (in % of potential yield) a = 0.01	a=0.0001
E-V	-Quadratic utility function -Risk averse insurer	α=0.05: Nov.-Sown. α=0.1: Oct.-Sown & Nov.-Sown	*53.0* 44.7 53.0	52.7 43.6 52.7	Dec.-Transplanted	3.4	3.2
FSD	-Positive marginal utility for wealth	Oct.-Sown & Nov.-Sown	44.7 53.0	43.6 52.7	Dec.-Transplanted	3.4	3.2
SSD	-Risk averse insurer	Oct-Sown	53.0	52.7	Dec.-Transplanted	3.4	3.2

Source: Author's calculations.

*Parameter a = 0.01, 0.0001 are the coefficients of risk aversion.

i. For maize production in Eldoret, Western Kenya, the *EV* criterion, at least for α=0.05, designates November as the appropriate time for sowing since the corresponding distribution has lower variance. Under *E-V* criterion, and for α=0.1, the two cannot be ranked. Figure 1 shows that under SSD criterion, October is the preferred sowing time that leads to higher yields, since the area beneath its cdf is smaller than that of November sowing. However, the two distributions cannot be ranked under FSD criterion because their two cdf cross each other.

ii. For rice crop near Antananarivo, Central High Plains of Madagascar, all three efficiency criteria indicate that transplantation of rice should take place in December if satisfaction of the water needs relies solely on rainfall (Figure 2).

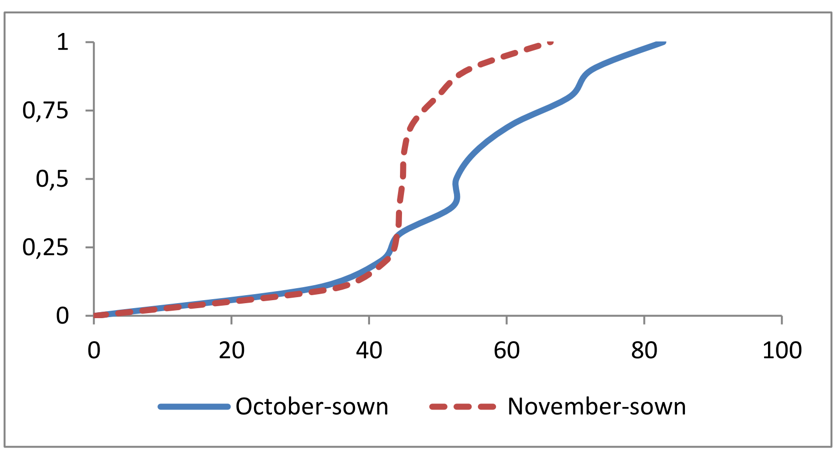

Figure 1. Cumulative Distribution Functions of Forecasted Maize Yields (in % of Potential Yield) for Two Sowing Dates

Source: Author' s estimations.

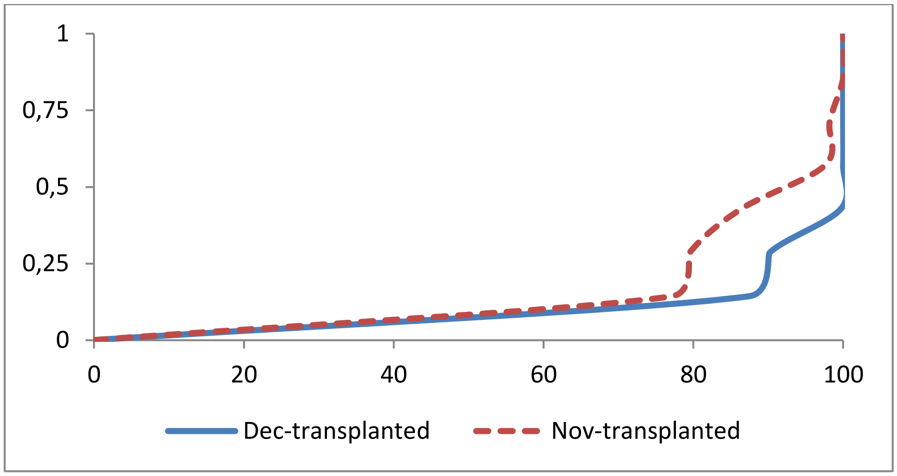

Figure 2. Cumulative Distribution Functions of Forecasted Rice Yields (in % of Potential Yield) for Two Transplantation Dates

Source: Author's estimations.

More important, Table 6 indicates that the values of the premium calculated for maize production at the two different values of risk aversion coefficient, are much higher compared to the premium values for rice. A risk averse insurer for maize production in Western Kenya would require a premium value between 43 and 55% of the potential (or normal) yields to fully cover the loss caused by lack of rainfall. Contrastingly, farmers in lowland rice in the Central High Plains of Madagascar may only pay less than 4 % of their yields per unit of land to insure against lack of rainfall. These results are robust to the changes in the risk aversion parameters of the insurer.

6. Conclusion and Discussion

This study aims to contribute to solving the difficult problem of determining crop insurance premium to deal with the risks due to drought. The method we presented shows that establishing a drought insurance implies investigating a complex relationship between the amount of rainfall on the one hand, and the crop's growth stage, and the soil's ability to retain humidity on the other hand. Premium estimation is based on forecasting yield loss due to water stress with a risk non-neutral insurer.

Implementing the proposed method on maize in Western Kenya and rice in Central High Plains of Madagascar, we find that the productivity of maize culture in Western Kenya would continue to suffer from the adverse impacts of lack of rainfall without establishing drought insurance. Higher productivity requires a continuing amelioration of the soil's texture and structure so that the soil can improve its ability to retain humidity and to make water available to crops when needed. But even if other conditions to produce good harvest (including fertilization, and pest control) were met, significant improvement in irrigation technology would still be required.

More important, as drought becomes more frequent in many places in Africa, irrigation technology has its limits and the use of risk management tools (such as drought and rainfall insurances) that help stabilize farm production and income is unavoidable. But the high estimates of premium values (more than half of the potential harvest) for maize show how insuring against drought can be costly. Efforts to help drought-stricken farmers afford such high premium value must be considered.

Contrastingly, lowland rice production in the Central High Plains of Madagascar still, at least during the observation periods, benefits from relatively favorable amount and distribution of rainfall and from the higher water retention of the lowland soil. Unfortunately, such a situation may be about to change. Rice farmers have lately complained that the rainy season has been more and more delayed, and often cumulated in a few months of heavy torrential rains spaced by weeks of high temperature and severe droughts. The worrying situation requires close monitoring of the variations in soil humidity and prompts the use of risk management tools.

The accuracy of our results can be improved in several ways. If data on rainfall, evapotranspiration, and other soil conditions are available at farm level or by field parcel location, throughout the year, a more precise calculation of differentiated premium values could be performed. Similarly, because farmers often grow diverse crops on the same land at the same time or at different periods of the year, a comprehensive premium package based on revenue loss per season due to drought has to be considered. Additionally, we employed non-parametric methods to compare and estimate some efficient sets of premium values, but when key data and parameters for longer periods are available, other methods such as time-series analysis and simulations to study the long run relationships among soil's conditions, rainfall amount and crop yields for premium estimation will be more effective.

When information on key variables (such as soil conditions, crop water needs, amount of rainfall and other climate related parameters) becomes more accessible to the public, the method we presented to set premium values shows that problems such as moral hazard and incomplete and asymmetric information can be overcome. With more accurate and transparent information on premium values, the availability and access to drought insurance benefitting both insurers and farmers will certainly increase.

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Appendix I

The utility of the insurer is the expected utility from his initial wealth, *x* and the increased wealth from the insurance activity, which is the amount of premium, *P*, subtracted by the indemnity *S*.

$$u(x) = E[u(x + P - S)] \tag{9}$$

At one point in time, and for *x*=0, a new function $\hat{u}(.)$ is introduced in order to represent solely the utility from the insurer's gain, which under the assumption of "fairness" is equal to the utility from the initial wealth, set arbitrarily to be zero.

$$\hat{u}(0) = E[\hat{u}(P - S)] = 0 \tag{10}$$

If the utility function is of the form $u(x) = (1 - e^{-ax}) / a$, then

$$\begin{aligned} &\hat{u}(0) = \frac{1}{a} E[1 - e^{-a(P-S)}] = 0 \\ &1 - e^{-aP} E[e^{aS}] = 0, \end{aligned} \tag{11}$$

Solving for *P*,

$$\begin{aligned} &P = \frac{1}{a} \ln(E[e^{aS}]), \\ &or, \\ &P = \ln(M(a)) / a \end{aligned} \tag{12}$$

where *M*(.) is the moment generating function of the random variable *S*.