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A New High Order Theory for Buckling Temperature Analysis of Functionally Graded Sandwich Plates Resting on Elastic Foundations

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Using a high order theory (HSDT), this work presents a study of thermal buckling of functionally graded (FG) sandwich plates subjected to various temperature rises across their thickness and resting on a two-parameter elastic foundation. The mechanical properties of the FG sandwich plates are supposed to change gradually through the thickness according to a power law (P-FGM). The intermediate layer is homogeneous and made of a purely ceramic material. The principle of virtual works is used to obtain stability equations, and their solutions are obtained based on Navier's solution technique. The obtained results are compared with other studies in the literature. Then, a parametric study is conducted to investigate the influence of geometric and mechanical characteristics such as the ratios of dimensions (width, length, and thickness), the material index (*k*), and the effect of the elastic foundation on the critical buckling temperature.

Keywords: FG sandwich plates, Thermal buckling, Elastic foundation, Virtual works, Navier's solution.

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1. INTRODUCTION

Functionally graded materials (FGM) are a class of composites, in which the properties of the material gradually change over one or more Cartesian directions [1], the combination of which results in an assembly with higher performance than components taken separately. This class of composite materials has gained considerable attention in the engineering community, especially in high-temperature applications such as nuclear reactors, aerospace, and power generation industries [2]. The advantages of continuously varying volume fraction of the constituent phases are eliminating high-stress concentrations and separating layers encountered in other composites such as sandwiches and laminates. Over the last two decades, there has been a significant number of reports on thermal stresses, fracture, thermo-mechanical response, buckling, free vibration, etc. of FGM structural elements [3].

Zenkour [4] investigated the response and bending, buckling, and free vibration of the simply supported sandwich plate using the plate theory of sinusoidal shear strain. Zhao [5] studied the buckling behavior of FG plates under mechanical and thermal loads with arbitrary geometry. Bourada [6] presented a novel theory of refined four-variable plates for the thermal buckling analysis of FG sandwich plates. Kettaf [7] examined the thermal buckling response of FG sandwich plates by proposing a new model of hyperbolic displacement. Bouiadjra [8] proposed a refined four-variable plate theory to analyze buckling of FG plates subjected to thermal loads. Maiche [9] using the refined hyperbolic shear strain plate theory had four variables for the study of buckling and vibration of FGM plates.

This work analyzes a new displacement model for sandwich plates on an elastic foundation under various thermal loads. Analytical solutions are presented using a New High Order Theory (HSDT).

2. GEOMETRY OF A FUNCTIONALLY GRADED PLATE

A simply supported FGM sandwich plate of length a, width b, and thickness h is referred to the rectangular cartesian coordinates (x, y, z), as shown in Fig. 1. The face layers of the FG sandwich plate are made of an isotropic material with material properties varying smoothly in the z direction only (FGM). The core layer is made of an isotropic homogeneous material (ceramic).

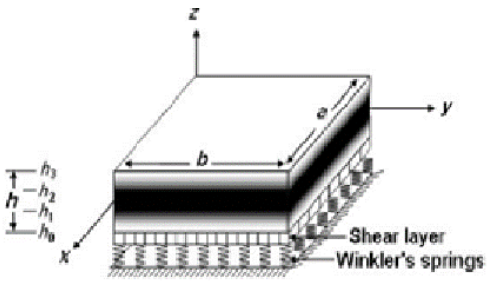

Fig. 1 – Geometry and coordinate system of the FGM sandwich plate resting on a two-parameter elastic foundation [10]

The effective material properties of FGM sandwich plates are expressed by [11]:

$$P^{(n)}(z) = P_m + (P_c - P_m)V^{(n)} . \qquad (2.1)$$

The volume fraction of FGM is supposed to obey a power-law function along the thickness direction

$$\begin{aligned} &v^{(1)} = (z - h_0 / h_1 - h_0)^k && h_0 \le z \le h_1 \\ &v^{(2)} = 1 && h_1 \le z \le h_2 \qquad (2.2) \\ &v^{(1)} = (z - h_3 / h_2 - h_3)^k && h_2 \le z \le h_3 \end{aligned}$$

where $V^{(n)}$ (n = 1, 2, 3) denotes the volume fraction

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function of layer n; k is the volume fraction index, which indicates the variation profile of the materials through the thickness.

3. DISPLACEMENT FIELD AND CONSTITUTIVE EQUATIONS

The displacement field of the P-FGM plate according to the high order theory is written in the following form:

$$\begin{aligned} &u(x,y,z,t) = u_0(x,y,t) - z(\partial w_0/\partial x) + k_1 f(z)\int \theta dx \\ &v(x,y,z,t) = v_0(x,y,t) - z(\partial w_0/\partial y) + k_2 f(z)\int \theta dy \\ &w(x,y,z,t) = w_0(x,y,t) \\ &\int \theta dx = A(\delta\theta/\delta x), \quad \int \theta dy = B(\delta\theta/\delta y) \end{aligned} \tag{3.1}$$

where u_0, v_0, w_0, θ are four unknown displacements of the mid-plane of the FG plate. The form of shape function $f(z)$ is given as follows:

$$f(z) = (3/25)\,\pi z\left(\pi - \sqrt[3]{0.135}\cosh\left(\pi z/h\right)\right). \tag{3.2}$$

Using the theory of elasticity, the strain components are obtained from equations (3.3) as follows:

$$\begin{aligned} &\varepsilon_{xx} = \varepsilon_{xx}^0 + zk_x^1 + f(z)k_x^2, \\ &\varepsilon_{yy} = \varepsilon_{yy}^0 + zk_y^1 + f(z)\mathrm{k}_y^2, \\ &\gamma_{xy} = \gamma_{xy}^0 + zk_{xy}^1 + f(z)k_{xy}^2, \\ &\gamma_{yz} = \partial f(z)/\partial z\ \gamma_{yz}^0, \\ &\gamma_{xz} = \partial f(z)/\partial z\ \gamma_{xz}^0, \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} &\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \partial u_0/\partial x \\ \partial v_0/\partial y \\ \partial u_0/\partial y + \partial v_0/\partial x \end{Bmatrix}, \quad \begin{Bmatrix} k_x^1 \\ k_y^0 \\ k_{xy}^1 \end{Bmatrix} = \begin{Bmatrix} -\partial^2 w_0/\partial x^2 \\ -\partial^2 w_0/\partial y^2 \\ -2\,\partial^2 w_0/\partial x\partial y \end{Bmatrix}, \\ &\begin{Bmatrix} k_x^2 \\ k_y^2 \\ k_{xy}^2 \end{Bmatrix} = \begin{Bmatrix} (k_1 A)\ \partial^2\theta/\partial x^2 \\ (k_2 B)\ \partial^2\theta/\partial y^2 \\ (k_1 A + k_2 B)\ \partial^2\theta/\partial x\partial y \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} = \begin{Bmatrix} k_1 A\ \partial\theta/\partial x \\ k_2 B\ \partial\theta/\partial y \end{Bmatrix}, \end{aligned} \tag{3.4}$$

The stresses in layer n can be expressed as follows:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix}^{(n)} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}^{(n)} \left(\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{Bmatrix} - \begin{Bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \alpha^{(n)}(\mathrm{z})\mathrm{T} \right) \tag{3.5}$$

where

$$\begin{aligned} &C_{11}^{(\mathrm{n})} = C_{22}^{(\mathrm{n})} = E^{(\mathrm{n})}(\mathrm{z})/1 - \left(\nu^{(\mathrm{n})}\right)^2, \ C_{12}^{(\mathrm{n})} = \nu^{(\mathrm{n})} C_{11}^{(\mathrm{n})} \\ &C_{44}^{(\mathrm{n})} = C_{55}^{(\mathrm{n})} = C_{66}^{(\mathrm{n})} = E^{(\mathrm{n})}(\mathrm{z})/2\left(1 + \nu^{(\mathrm{n})}\right) \end{aligned} \tag{3.6}$$

4. STABILITY EQUATIONS

The governing equations are obtained using the principle of virtual displacement, which equates external and internal work. The total potential energy of the system V (FG plate resting on the Pasternak elastic foundation) may be written in the form [12]:

$$V = U + U_F. \tag{4.1}$$

Here U is the total strain energy of the plate, which is calculated as

$$U = 1/2\left(\int_V \begin{bmatrix} \sigma_{xx}^{(\mathrm{n})}\left(\varepsilon_{xx} - \alpha^{(\mathrm{n})}T\right) + \sigma_{yy}^{(\mathrm{n})}\left(\varepsilon_{xx} - \alpha^{(\mathrm{n})}T\right) \\ + \sigma_{xy}^{(\mathrm{n})}\gamma_{xy} + \sigma_{yz}^{(\mathrm{n})}\gamma_{yz} + \sigma_{xz}^{(\mathrm{n})}\gamma_{xz} \end{bmatrix} dV \right)^{(n)}, \tag{4.2}$$

U_F is the strain energy due to the Pasternak elastic foundation, which is given by [13]:

$$U_F = 1/2\left(\int_\Omega K_w w_0^2 + K_g\left(w_{0,x}^2 + w_{0,y}^2\right) d\Omega \right). \tag{4.3}$$

Substituting Eq. (3.4) and Eq. (3.5) into Eq. (4.2) and integrating through the thickness of the plate, Eq. (4.2) can be rewritten as:

$$\begin{aligned} &\int_\Omega \begin{bmatrix} N_{xx}\delta\varepsilon_{xx}^0 + N_{yy}\delta\varepsilon_{yy}^0 + N_{xy}\delta\gamma_{xy}^0 + M_{xx}\delta k_x^1 \\ + M_{yy}\delta k_y^1 + M_{xy}\delta k_x^1 + P_{xx}\delta k_x^2 + P_{yy}\delta k_x^2 \\ + P_{xy}\delta k_x^2 + Q_{yz}\gamma_{yz}^0 + Q_{yz}\gamma_{xz}^0 \end{bmatrix} d\Omega \\ &- \int_\Omega \left[K_w w_0^2 + K_g\left(w_{0,x}^2 + w_{0,\mathrm{y}}^2\right) \right] d\Omega = 0 \end{aligned} \tag{4.4}$$

where

$$\begin{aligned} &\begin{bmatrix} N_x & N_y & N_{xy} \\ M_x & M_y & M_{xy} \\ P_x & P_y & P_{xy} \end{bmatrix} = \sum_{n=1}^{3} \int_{h_n}^{h_{n+1}} \left(\sigma_x, \sigma_y, \tau_{xy}\right)^{(\mathrm{n})} \begin{Bmatrix} 1 \\ z \\ f(\mathrm{z}) \end{Bmatrix} dz, \\ &\left(Q_{xz}, Q_{yz}\right) = \sum_{n=1}^{3} \int_{h_n}^{h_{n+1}} \left(\tau_{xy}, \tau_{xy}\right)^{(\mathrm{n})} \partial f(z)/\partial z\, \mathrm{d}z \end{aligned} \tag{4.5}$$

h_n and h_{n+1} are the bottom and top z-coordinates of the n-th layer.

Substituting Eq. (3.3) and Eq. (3.5) into Eq. (4.5), the relationships between the stress resultants and the strains are expressed in the matrix form as:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \\ P_{xx} \\ P_{yy} \\ P_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & C_{11} & C_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & C_{12} & C_{2} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & C_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & F_{11} & F_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & F_{12} & F_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & F_{66} \\ C_{11} & C_{12} & 0 & F_{11} & F_{12} & 0 & H_{11} & H_{12} & 0 \\ C_{12} & C_{22} & 0 & F_{12} & F_{22} & 0 & H_{12} & H_{22} & 0 \\ 0 & 0 & C_{66} & 0 & 0 & F_{66} & 0 & 0 & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{xx} \\ \gamma^0_{xy} \\ k^1_x \\ k^1_y \\ k^1_{xy} \\ k^2_x \\ k^2_y \\ k^2_{xy} \end{Bmatrix} - \begin{Bmatrix} N^T_{xx} \\ N^T_{yy} \\ 0 \\ M^T_{xx} \\ M^T_{yy} \\ 0 \\ P^T_{xx} \\ P^T_{yy} \\ 0 \end{Bmatrix}, \quad (4.6)$$

$$\begin{Bmatrix} Q_{yz} \\ Q_{yz} \end{Bmatrix} = \begin{bmatrix} A_{44} & 0 \\ 0 & A_{55} \end{bmatrix} \begin{Bmatrix} y^0_{yz} \\ y^0_{zz} \end{Bmatrix}, \quad (4.7)$$

where the stiffness components are given as follows:

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & C_{11} & F_{11} & H_{11} \\ A_{12} & B_{12} & D_{12} & C_{12} & F_{12} & H_{12} \\ A_{66} & B_{66} & D_{66} & C_{66} & F_{66} & H_{66} \end{Bmatrix} = \sum_{n=1}^{3} \int_{h_n-1}^{h_n} C_{11}[1, z, z^2, f(z), z f(z), f^2(z)] \begin{Bmatrix} 1 \\ \nu \\ (1-\nu)/2 \end{Bmatrix} dz\,, \quad (4.8)$$

$$\begin{aligned} &\left(A_{22}, B_{22}, D_{22}, C_{22}, F_{22}, H_{22}\right) = \left(A_{11}, B_{11}, D_{11}, C_{11}, F_{11}, H_{11}\right) \\ &A_{44} = A_{55} = \sum_{n=1}^{3} \int_{h_n-1}^{h_n} C_{44}\left[df(z)/\mathrm{d}z\right]^2 dz \end{aligned} \quad (4.9)$$

The thermal force and thermal moment resultants, $N^T_x = N^T_y, M^T_x = N^T_y$ and $P^T_x = P^T_y$ are defined by:

$$\begin{Bmatrix} N^T_x \\ M^T_x \\ P^T_x \end{Bmatrix} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} E^{(n)}(z)\left(1-\nu^{(n)}\right)\alpha^{(n)}(z)\,\mathrm{T} \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \quad (4.10)$$

Substituting δU and δU_F into the virtual work statement (4.1), integrating by parts and collecting the coefficients δu_0, δv_0, δw_0, and $\delta\theta$, the following governing equations are obtained:

$$\begin{aligned} &\partial N^1_{xx}/\partial x + \partial N^1_{xy}/\partial y = 0 \\ &\partial N^1_{xy}/\partial x + \partial N^1_{yy}/\partial y = 0 \\ &\partial^2 M^1_{xx}/\partial x^2 + \partial^2 M^1_{xy}/\partial x \partial y + \partial M^1_{yy}/\partial y^2 + \bar{N} - K_w w^1_0 + K_w \nabla^2 w^1_0 = 0 \\ &-(K_1 A)\,\partial^2 P^1_{xx}/\partial x^2 - (K_1 A + K_2 B)\,\partial^2 P^1_{xy}/\partial x \partial y - (K_2 B)\,\partial P^1_{yy}/\partial y^2 \\ &\qquad + (K_1 A)\,\partial^2 Q^1_{xz}/\partial x \partial z + (K_2 B)\,\partial^2 Q^1_{yz}/\partial y \partial z = 0 \end{aligned} \quad (4.11)$$

with

$$\bar{N} = \left[\bar{N}^0_{xx}(\partial^2 w^1_{00}/\partial x^2) + \bar{N}^0_{yy}(\partial^2 w^1_0/\partial y^2)\right]\upsilon\,, \quad (4.12)$$

where

$$N^0_{xx} = N^0_{yy} = -\sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} E^n(z)\alpha^n(z)T/\left(1-\nu^n\right) dz\,. \quad (4.13)$$

5. ANALYTICAL SOLUTION FOR SIMPLY SUPPORTED RECTANGULAR FGM SANDWICH PLATES

Rectangular plates are generally classified according to the type of support. For a simply supported rectangular FG sandwich plate, the Navier method, based on double Fourier series, is used for the analytical solution of the partial differential equation

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \begin{Bmatrix} U\cos(\lambda x)\sin(\mu y) \\ V\sin(\lambda x)\cos(\mu y) \\ W\sin(\lambda x)\sin(\mu y) \\ X\sin(\lambda x)\sin(\mu y) \end{Bmatrix} \quad (5.1)$$

where $\lambda = m\pi/a$, $\mu = n\pi/y$; U, V, W and X are arbitrary parameters to be determined. Substitution of the stress and moment resultants defined in Eq. (4.6) and Eq. (4.7) into equations (4.11) gives:

$$\left[L\right]\left\{Q\right\} = 0 \quad (5.2)$$

where

$$\left\{\Omega\right\} = \left\{U, V, W, X\right\}^T \quad (5.3)$$

and $[L]$ is the symmetric matrix given by:

$$[L]=\begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{12} & L_{22} & L_{23} & L_{24} \\ L_{13} & L_{23} & L_{33} & L_{33} \\ L_{14} & L_{24} & L_{33} & L_{44} \end{bmatrix}, \quad (5.4)$$

in which:

$$\begin{aligned}
&L_{11} = A_{11}\lambda^2 + A_{66}\mu^2, \quad L_{12} = (A_{12}+A_{66})\lambda\mu \\
&L_{13} = -B_{11}\lambda^3 - (B_{12}+2B_{66})\lambda\mu^2 \\
&L_{14} = Cs_{11}K_1A\lambda^3 + (Cs_{12}K_2B \\
&\quad + Cs_{66}(K_1A+K_2B))\lambda\mu^2 \\
&L_{22} = A_{11}\mu^2 + A_{66}\lambda^2, L_{23} = -B_{22}\mu^3 - (B_{12}+2B_{66})\lambda^2\mu \\
&L_{23} = (Cs_{12}K_1A + Cs_{66}(K_1A+K_2B))\lambda^2\mu + Cs_{11}K_2B\mu^3 \\
&L_{33} = -D_{11}\lambda^4 - D_{22}\mu^4 - 2(D_{12}+2D_{66})\lambda^2\mu^2 \\
&\quad - N^0_{xx}\lambda^2 + N^0_{yy}\mu^2 - k_w - k_g(\lambda^2+\mu^2) \\
&L_{34} = F_{11}K_1A\,\lambda^4 + (F_{12}+2F_{66})(K_1A+K_2B))H_{22} \\
&\quad + F_{22}K_2B\mu^4 \\
&L_{44} = -(2H_{12}K_1A\,K_2B + H_{66}(K_1A+K_2B)^2)H_{22} \\
&\quad - H_{11}(K_1A)^2\lambda^4 - H_{22}(K_2B)^2\mu^4 + A_{44}((K_1A)^2\lambda^2 \\
&\quad + (K_2B)^2\mu^2)
\end{aligned} \quad (5.5)$$

6. THERMAL BUCKLING OF FGM PLATES

In the following, a square plate subjected to thermal loads is considered. To examine the effect of the type of temperature variation, three types of thermal loading in the thickness of the plate are taken.

6.1 Buckling of FGM Sandwich Plates under Uniform Temperature Rise

The initial uniform temperature of the plate is assumed to be T_i. The temperature is uniformly raised to a final value T_f in which the plate buckles. The temperature change is [14]:

$$T(z) = T_f - T_i = \Delta T\ . \quad (6.1)$$

6.2 Buckling of FGM Sandwich Plates under Linear Temperature Rise (LTR)

For FGM sandwich plates, the temperature change is not uniform, the temperature distribution is approximated linearly through the thickness. Therefore, the temperature as a function of the thickness coordinate z can be written in the form [15]

$$T(z) = \Delta T(z/h + 1/2) + T_M,\ \Delta T = T_C - T_M\ . \quad (6.2)$$

6.3 Buckling of FGM Sandwich Plates under Non-Linear Temperature Rise

We assume that the temperature varies from T_t, according to the power-law variation through the thickness, to the bottom surface temperature T_b, in which the plate buckles. In this case, the temperature through the thickness is given by

$$T(z) = \Delta T(z/h + 1/2)^p + T_m,\ \Delta T = T_C - T_m, \quad (6.3)$$

where T_C and T_M are the temperatures of the top and bottom surfaces.

7. RESULTS AND DISCUSSION

In this section, numerical results are presented for the static analysis of FGM sandwich plates resting on elastic foundations and subjected to thermal loading. The FG sandwich plate is taken to be made of Titanium and Zirconia with the following material properties: [6]: E_m = 66.2 GPa, $\alpha_m = 10.3\cdot 10^{-6}$ 1/c°, K_m = 18.1 W/m.K, E_c = 244.27 GPa, $\alpha_c = 12.766\cdot 10^{-6}$ 1/c°, K_c = 1.7 W/m.K.

Poisson's ratio is selected constant for both Titanium and Zirconia and it equals to 0.3. Numerical results are presented in terms of non-dimensional thermal buckling. The various non-dimensional parameters used are [16]:

$$kw = Kwa^4/D_c,\ kg = Kga^2/D_c,\ D_c = E_ch^3/12(1-\nu^2). \quad (7.1)$$

7.1 Validation of the Results

In order to validate the accuracy of the present formulations, a comparison has been carried out with the results obtained in [6], [17] and [18], based on the higher-order shear deformation theories (HSDT). The results are listed in Table 1, Table 2 and Table 3.

Table 1 – Critical buckling temperature T_{cr} of square FG plate under uniform temperature rise for different values of power law index k and side-to-thickness ratio a/h

a/h	**Theory**	**05**			**10**		**50**	
k		**0.5**	**2**	**10**	**0.5**	**2**	**0.5**	**2**
1-1-1	**Present**	2.83224	2.35820	2.19000	0.79458	0.64226	0.03308	0.02645
	[6]	2.83224	2.36000	2.19469	/	/	/	/
	[17]	2.80230	2.31737	/	0.79456	0.64238	0.03308	0.02645
1-0-1	**Present**	2.86962	2.62851	3.29414	0.80306	0.71749	0.03340	0.02958
	[6]	2.87074	2.63018	3.30340	/	/	/	/
	[17]	2.83507	2.57355	/	0.80313	0.71783	0.03340	0.02958
2-1-2	**Present**	2.82961	2.39307	2.41453	0.79216	0.65051	0.02676	0.02676
	[6]	2.83030	2.39637	2.42186	/	/	/	/
	[17]	2.79675	2.34734	/	0.79220	0.65075	0.02676	0.02676

Table 2 – Critical buckling temperature T_{cr} of FGM sandwich square plates under non-linear temperature rise for different values of power law index k and side-to-thickness ratio a/h

a*/*h	**Theory**	**05**			**10**		**50**	
k		**0.5**	**2**	**10**	**0.5**	**2**	**0.5**	**2**
1-1-1	**Present**	21.1247	21.9657	22.0600	5.79104	5.81198	0.06079	0.01363
	[6]	21.1243	21.9830	22.1070	/	/	/	/
	[17]	21.1243	21.9830	/	5.79091	5.81247	0.06078	0.01363
1-0-1	**Present**	21.6048	22,9880	23,9434	5.90994	6.12150	0.06380	0.04051
	[6]	21.6133	23.0292	24.0112	/	/	/	/
	[17]	21.6133	23.0292	/	5.90995	6.12449	0.06380	0.04052
2-1-2	**Present**	21.3330	22.3216	22.7434	5.83535	5.89614	0.06048	0.01668
	[6]	21.3382	22.3527	22.8131	/	/	/	/
	[17]	21.3382	22.3527	/	5.83566	5.89838	0.06048	0.01668

Table 3 – The effects of elastic foundation parameters on the critical buckling temperature difference T_{cr} of FGM sandwich square plates under thermal load (k = 2, a/h = 50)

Schema	**(1-2-1)**			**(1-1-1)**			**(2-1-2)**		
kg = 10	**kw**			**kw**			**kw**		
	10	50	100	10	50	100	10	50	100
Present	35.37540	36.88610	38.77450	35.38110	37.10400	39.25770	36.83370	38.77480	41.20120
[18]	35.37104	36.88174	38.77011	35.37592	37.09886	39.25252	36.82788	38.76898	41.19535
kw = 10	**kg**			**kg**			**kg**		
	5	10	25	5	10	25	5	10	25
Present	31.64790	35.37540	46.55790	31.12990	35.38110	48.13460	32.04420	36.83370	51.20210
[18]	31.64354	35.37104	46.55352	31.12475	35.37592	48.12944	32.03841	36.82788	51.19629

7.2 Parametric Studies

The influence of the elastic foundation on the critical buckling temperature of FGM sandwich plates (2-1-2) is shown in Fig. 2. As observed, for plates with/without elastic foundation, we can see in these figures that, whatever the type of thermal loading and the values of the constant of the elastic foundation (kw and kg), the critical buckling temperature of the sandwich plate decreases as the aspect ratio a/b increases for all values of the elastic foundation. The critical buckling temperature is almost constant when $b/a > 2$.

The variation of critical temperatures T_{cr} of FGM sandwich plates subjected to various thermal loading types (UTR, LTR, and NTR) versus the side-to-thickness ratio a/h is shown in Fig. 3. For the four types of plates with a small thickness ratio, considerable differences between the results for different temperature variations are observed. However, for a significant value of the thickness ratio, the difference between the critical temperature values is slight. It is clear and regardless of the type of loading or the type of the sandwich plate. It is observed that the critical buckling temperature decreases with an increase in the side-to-thickness ratio a/h.

Fig. 4 shows the effect of the inhomogeneity parameter k on the critical buckling temperature T_{cr} for deferent types of FGM sandwich plates under various thermal loadings. It can also be seen that the critical temperature T_{cr} under uniform temperature rise is smaller than that of the plate under linear temperature rise, and the latter is smaller than that of the plate under nonlinear temperature rise.

It is noted that for the plate without a core or the core of the plate is half of the thickness of the face in the case of sandwiches (1-0-1) and (2-1-2), the critical buckling temperature T_{cr} decreases to reach minimum values and then increases gradually as the inhomogeneity parameter k increases. However, for the (1-1-1) and (1-2-1) FGM sandwich plates, the critical buckling temperature T_{cr} decreases smoothly as the power-law index k increases.

In Fig. 5 and Fig. 6, we see that the elastic foundation parameters have little effect on the value of critical buckling temperature, an increase in the shear modulus parameter leads to an increase in the critical buckling temperature.

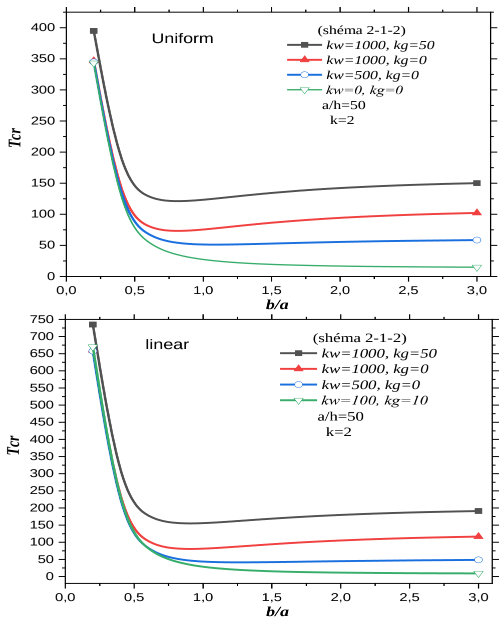

Fig. 2 – Effect of the aspect ratio (a/b) and parameters of elastic foundations on the critical buckling temperature T_{cr} of a FGM sandwich plate simply supported (k = 2, a/h = 50)

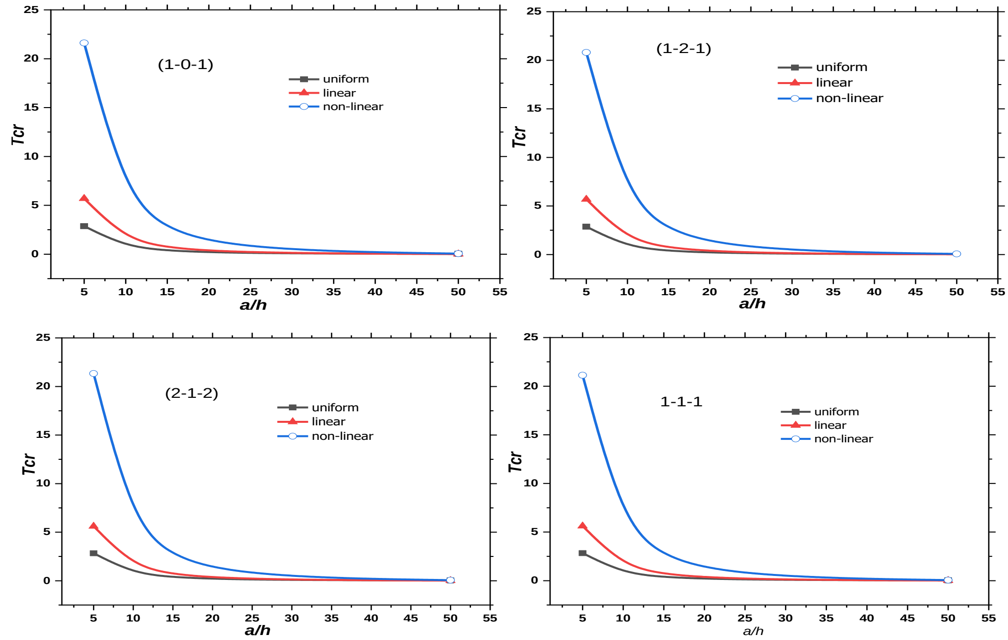

Fig. 3 – Variation of the critical buckling temperature versus the side-to-thickness ratio a/h for different types of FGM sandwich plates

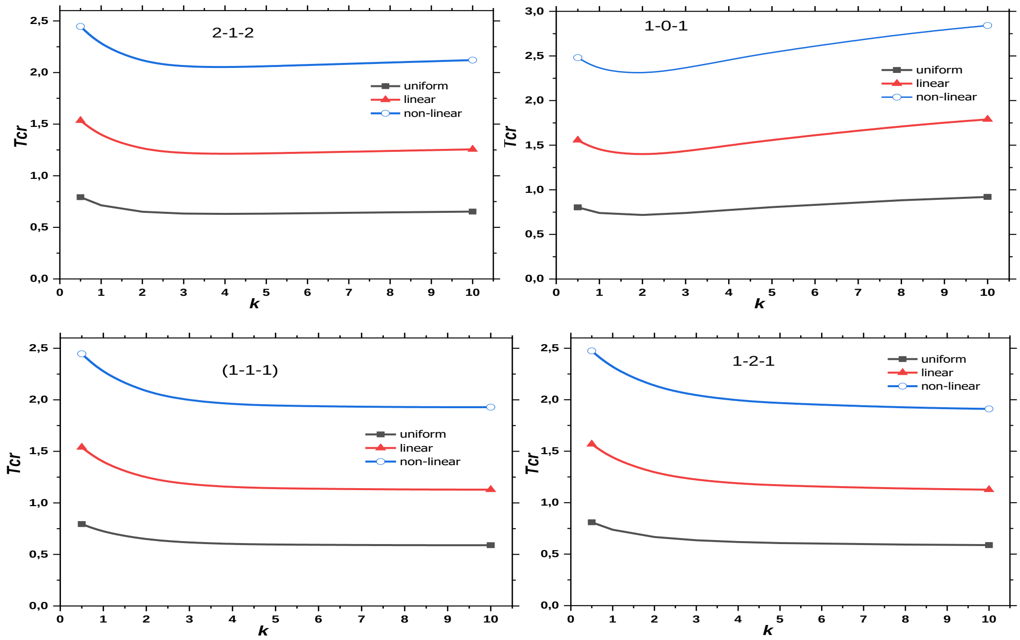

Fig. 4 – Critical buckling temperature difference T_{cr} due to the uniform, linear and nonlinear temperature rise across the thickness versus the power-law index k ($a/h = 10$)

8. CONCLUSIONS

In the present study, thermal buckling analysis of FGM sandwich plates resting on an elastic foundation has been performed. The present method is based on a 2D plate shear deformation theory that contains four unknowns. Critical buckling loads of FGM sandwich plates have been analyzed using an analytical approach. Based on the higher-order shear deformation theories, FGM sandwich plate buckling analyses were carried out considering a sandwich plate under various thermal loadings. The obtained results were presented in figures and tables and compared with references, demonstrating the accuracy of the current approach. Based on the results obtained, the following conclusions can be drawn.

By increasing the thickness of the core (the ceramic layer) of sandwich plates, the critical buckling temperature difference decreases.

The plate's geometry also influences the critical temperature: the critical buckling temperature difference of FGM sandwich plates decreases when the side-to- thickness ratio a/h increases.

By increasing the value of the material index, the critical temperature initially decreases and then

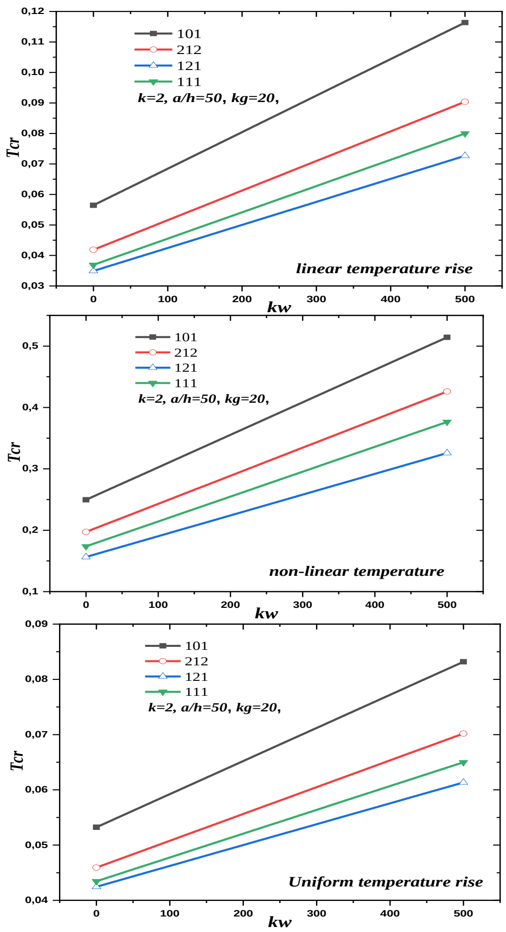

Fig. 5 – Variation of the critical buckling temperature versus the para meter kw (a/h = 50) for different types of FGM sandwich plates under uniform, linear and nonlinear temperature rise

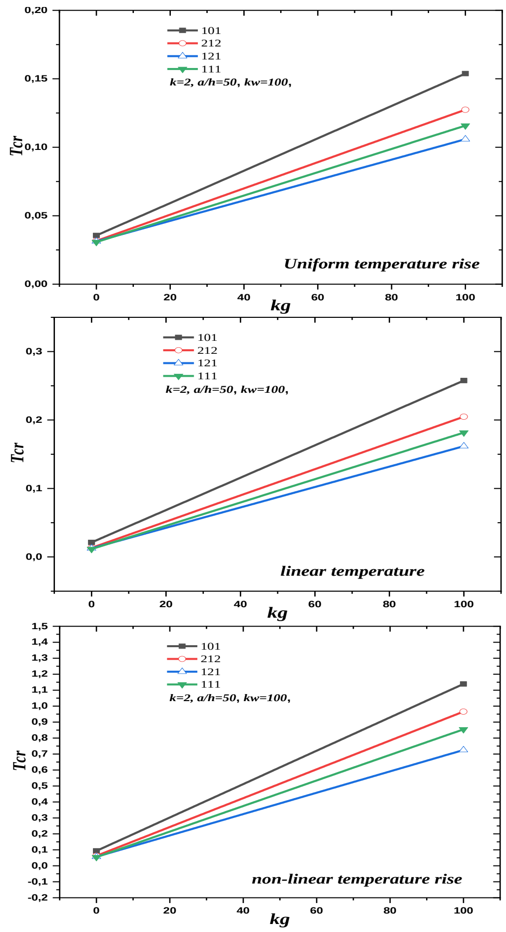

Fig. 6 – Variation of the critical buckling temperature versus the parameter kw (a/h = 50) for different types of FGM sandwich plates under uniform, linear and nonlinear temperature rise

$h_{core} < h_{face}$, but for structures where $h_{core} \geq h_{face}$ the critical temperature T_{cr} decreases gradually to minimum values. The critical buckling temperature difference T_{cr} for FG plates increases with increasing the elastic foundation parameter. The rigidity of the foundation has a significant effect on the buckling temperature of sandwich plates. The effect of kg on the critical buckling temperature is more pronounced than kw.

In conclusion, it can be said that the proposed theory is accurate and simple in solving thermal buckling behaviors of functionally graded sandwich plates resting on an elastic foundation.

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Нова теорія високого порядку для аналізу температури жолоблення функціонально градуйованих сендвіч-пластин, що спираються на пружну основу

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Використовуючи теорію високого порядку (HSDT), у роботі представлено дослідження термічного жолоблення функціонально градуйованих (FG) сендвіч-пластин, підданих різному підвищенню температури по всій їх товщині, що спираються на двопараметричну пружну основу. Передбачається, що механічні властивості FG сендвіч-пластин поступово змінюються з товщиною відповідно до степеневого закону (P-FGM). Проміжний шар однорідний і виготовлений з чисто керамічного матеріалу. Принцип віртуальних робіт використовується для отримання рівнянь стійкості, розв'язки яких отримують на основі методики розв'язування рівнянь Нав'є. Отримані результати порівнюються з результатами інших досліджень. Потім проводиться параметричне дослідження для вивчення впливу геометричних і механічних характеристик, таких як співвідношення розмірів (ширина, довжина і товщина), індекс матеріалу (k) та вплив пружної основи на критичну температуру жолоблення.

Ключові слова: FG сендвіч-пластини, Термічне жолоблення, Пружна основа, Віртуальні роботи, Рішення Нав'є.