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NEW TRENDS IN MARKET RISK MANAGEMENT

This paper concentrates on the methodology of constructing the fuzzy knowledge base of the investment process with the support of the fuzzy set theory. Fuzzy sets, fuzzy numbers and linguistic variables have been used here as indispensable tools to build the linguistic models which help formalize both objective and expert knowledge. These particular models can be applied for forecasting purposes such as fuzzy investment process prediction.

Key words: fuzzy knowledge, risk management, investment process, fuzzy sets, fuzzy numbers and linguistic variables.

Introduction. Investment, interpreted as the transformation of capital in the hope of achieving a particular objective, will generate events whose development and whose mechanisms are difficult or impossible to predict. This fact, coupled with the inability to precisely determine the future states of the environment in which the investment process[1] takes place, makes the achievement of an investment objective uncertain. Due to the unique nature of capital markets, capital investments are perhaps where this uncertainty is the most conspicuous, that is, where one can be the least certain of achieving one's investment objectives.

The following distinction is commonly made in subject literature (Tarczyński [2003]):

- measurable uncertainty – risk;
- immeasurable uncertainty – uncertainty proper, or *sensu stricte*.

Uncertainty is said to be measurable if the following conditions are met:

- the future states of the environment can be identified;
- the probability distributions of the future environment states are known.

Considering this, it only makes sense to speak of managing risk, while uncertainty *sensu stricte* will remain an open question.

The conditions mentioned above for uncertainty to be measurable will produce a data base providing for identification of the probability characteristics of the investment process that are instrumental in building quantitative models of risk management. It must be emphasized that the investment process data base comprises objective knowledge of the environment states (knowledge acquired through empirical research) and the *a priori* assumptions on the mechanisms governing the process. This means that investment risk management will always be conditional, relying on a number of conditions.

Capital investment risk can be regarded in terms of a dichotomy: it will have a positive aspect attributed to the opportunity of realizing extraordinary profits, but it will only be so if the risk is efficiently managed; otherwise, it will be considered to have clearly negative implications. As a precondition for the efficient management of investment risk, it is necessary to quantify the risk and to identify all the environment factors involved. The possibility to satisfy these conditions is limited by our knowledge on the investment process contained in the data base as well as by our ability to utilize the data in quantifying the risk and identifying the environmental factors.

Besides objective knowledge, the capital investment practice has accumulated vast resources of expert knowledge. However, this potential is not usually exploited in building quantitative risk management models.

The incorporation of expert knowledge, alongside objective knowledge, into risk management models should contribute to improving their adequacy and precision, thus enhancing their efficiency.

Formalized methods for dealing with objective knowledge in investment process modeling are supplied by the probability calculus, whereas the fuzzy set theory allows the possibility to formalize expert knowledge.

Further in the paper, a methodology will be presented for processing both objective knowledge and expert knowledge.

Fuzzy Logic

The fuzzy set theory makes it possible to formalize the expert knowledge. The theory key concepts are: fuzzy sets, fuzzy numbers, linguistic variables. These particular concepts are defined

[1] In this paper, the investment process is viewed as a stochastic process expressing the value of a financial object.

within a certain universe of discourse **X** by a membership function.

The fuzzy set A we define as follows (Zadeh [1968]):

$$A = \{ (x, \mu_A(x)) : x \in X, \mu_A(x) \in <0, 1> \}, \quad (1)$$

where $\mu_A : X \to <0, 1>$ is a membership function of the membership of an element of the universe of discourse X to set A.

Let us assume that A is a set of high stock returns and the space X denotes all possible shares in the capital market. If one share has a noticeably high rate of return then its degree of membership to set A is 1. Consequently, if no growing tendency is observed in terms of the stock rate of return, the membership grade of such rate equals 0. The membership grade of a transitory share to set A is a number in the range of (0,1) which gets automatically closer to 1 once its characteristics get closer to the above mentioned set. In other words, the higher the rate of return, the closer the share gets to 1.

The set A is a fuzzy set fully characterized by the membership function.[2]

The accurate attribution of the membership grade to a set by the element of the universe of discourse discussed above is quite difficult. This operation is mostly subjective and contextualized.

It is important to keep in mind that the results of our discussions here as based on the theory of fuzzy sets are determined by the adequate defining of the membership function. In practice, the membership function is defined by means of either statistical survey method or an expert method. The latter is a method where an expert marks out general parameters of the membership function and the parameters of the function of a certain category are subsequently outlined by test and trial. More specifics on the nature of the membership function as well as algebra of fuzzy sets may be found in *A*. Lachwa's work (2001).

Fuzziness and probability which are phenomena of different nature and form may occur next to each other. According to Zadeh, a fuzzy random event is a fuzzy set defined within the domain of elementary events, measurable in Borel's terms.

The probability of the fuzzy random event A can be depicted in the form of the following equation:

$$P(A) = \sum_{\omega} p(\omega)\, \mu_A(\omega), \quad (2)$$

where $p\ (\omega)$ is the probability of the elementary event ω.

Fuzzy numbers, which are another important concept, can be defined as follows:

A fuzzy number is a normalized, convex fuzzy set outlined within the domain of real numbers R whose membership function is segmentally continuous.[3] Specifically, the LR fuzzy number is the fuzzy set A defined within the domain of real numbers outlined by the following membership function:

$$\mu_A(x) = \begin{cases} L_{(\frac{m-x}{\alpha})} & for\, x < m \\ 1 & for\, x = m \\ R_{(\frac{x-m}{\beta})} & for\, x > m \end{cases}, \quad (3)$$

where $L(.)$ – increasing function;
$R(.)$ – decreasing function;
α, β – positive parameters.

If X is a rate of return of a share, m – desirable value of the rate of return and $\alpha=\beta$ is equal to the standard deviation of the rate of return then the fuzzy number (2.3) represents the variability of the stock returns.

Linguistic variables are marked out by fuzzy sets, e.g. low, medium, high. We say that the stock rate of return is low, medium, high, which means that it is compatible with a certain range of real numbers where these numbers reflect the variability of the stock rate of return. The membership of an element to the fuzzy set (membership function) that specifies linguistic variables reflects the range of possibilities of this particular function. The above mentioned concepts can serve as tools to build fuzzy knowledge base for the investment process.

Let us assume that the investment process discussed thus far $X_t \in \boldsymbol{X} \subset R$, t=1,2… is the Markov process, i.e. the process which observes the following rule:

$$P\ (X_t / X_{t-1}, X_{t-2}, \ldots X_{t-k}) = P\ (X_t / X_{t-1}). \quad (4)$$

Let us assume that:

[2] The characteristic function which takes only two values 0 or 1 is a special type of a membership function.

[3] For the definition of a normalized, convex fuzzy set see A. Lachwa (2001).

$$X = \bigcup_{i=1}^{k} a_i ,\qquad (5)$$

where a_i are disjoint subsets of the universe of discourse X. Furthermore, let us suppose that the probability distribution of the investment process is known[4]:

$$p_{ij}(x_{t-1}, x_t) = P(X_{t-1} \in a_i , X_t \in a_j) \qquad (6)$$

$i, j = 1... k$

The (6) is used in order to determine boundary distribution of the process

$$p_{i\cdot}(x_{t-1}) = \sum_j p_{ij}(x_{t-1}, x_t) \qquad (7)$$

as well as its conditional distributions

$$p_{j/i}(x_t / x_{t-1}) = \frac{p_{ij}(x_{t-1}, x_t)}{p_i(x_{t-1})}) \;. \qquad (8)$$

The investment process will be defined here by linguistic categories as described below:

- within the universe of discourse X we outline l fuzzy states $A_1, A_2, \ldots A_l$ which represent the degree of the investment process (linguistic variables);
- we specify membership functions $\mu_{Aj}(x_t \in a_i) = \mu_{Aj}(a_i)$ that meet the condition of $\sum_j \mu_{Aj}(a_i) = 1$.

The probability of the fuzzy state A_j taking place as in (2) we can mark out with the aid of the following formula:

$$P(X_{t-1} \text{ is } A_j) = \sum_i p_i(x_{t-1})\, \mu_{Aj}(a_i) = P_{Aj}(x_{t-1}). \qquad (9)$$

The probability of A_i, A_j occurring jointly in relation to the investment process in t-1 and t we can define mathematically:

$$P(X_{t-1} \text{ is } A_i , X_t \text{ is } A_j) =$$

$$= \sum_{r=1}^{k}\sum_{s=1}^{k} p_{rs}(x_{t-1}, x_t)\mu_{Ai}(a_r)\mu_{Ai}(a_s) \cdot \qquad (10)$$

[4] The distribution of the investment process is outlined empirically on the basis of implementing the process X_t.

The formulas (9) and (10) make up the fuzzy knowledge base of the investment process.

Linguistic models

Linguistic model of the investment process.

Linguistic models are used for fuzzy forecasts of financial series. The following holds for the linguistic model:

$R^{(i)}$:{*IF(antecedent) THEN (consequent)*} $i = 1,2... p$,

where the antecedent describes a set of conditions whereas the consequent makes a conclusion (Helendoorn, Driankov (1997)).

To specify the linguistic model it is necessary to determine input variables (antecedent) as well as output variables (consequent), both of which are usually linguistic variables. Importantly enough, at this stage it is essential to determine fuzzy sets of these particular linguistic variables as well as, even more importantly, to outline their membership function.

The MIMO (multiple input—multiple output) model consists of the fuzzy rules of the following type:

$$R(i):\{wi \text{ IF } (x1 \text{ is } A1i \text{ i} \ldots \text{ i } xn \text{ is } Ani) \text{ THEN } (y_1 \text{ is } B_1^i \text{ i} \ldots \text{ i } y_m \text{ is } B_m^i)\}\ \ i=1...p, \qquad (11)$$

where w_i – weight of the rule;
$x = (x_1 ... x_n)$ – input variable, $x \in \mathbf{X} \subset R^n$;
$A_1^i ... A_n^i$ – linguistic values of the input variable;
$y = (y_1 ... y_m)$ – output variable, $y \in \mathbf{Y} \subset R^m$;
$B_1^i ... B_m^i$ – linguistic values of the output variable.

If inputs and outputs are independent variables then the MIMO model can be transformed in to the set of the SISO models (single input – single output). The fuzzy rules in the SISO model are as follows:

$$R^{(i)}\{ w_i \text{ IF } (x \text{ is } A_i) \text{ THEN } (y \text{ is } M_i)\}, \qquad (12)$$

where I – number of the fuzzy rule associated with the linguistic value A_i of the variable x;
M_i – structure of the consequent containing linguistic variable and weight.

The structure M_i can take the following form:

y is B_1^i with weight w_{i1}
also y is B_2^i with weight w_{i2}
....
also y is B_m^i with weight w_{im}

The weights w_{ij} specify the linguistic model significantly; they are denoted either statistically or by experts.

The model (12)[5] can be used for fuzzy forecasts of the investment process. In this case, we replace the weight w_i with the probability $P_{Ai}(x_{t-1})$ and define the weights w_{ij} by the use of the conditional probabilities $P_{Ai/Aj}(x_t/x_{t-1})$. The input variable is the value of the investment process in $t-1$ whereas the output variable is the value of the process in t. The fuzzy sets A_i and B_i are identical to the fuzzy states of the process as it was discussed in the second part of this paper.

Quasi VaR

The quantification of risk relates directly to risk measurement. Risk measures can be classified in the following groups:

- volatility measures;
- downside risk measures;
- sensitivity measures.

Each risk measure has its specific functions and an established range of applications. All of them, however, in one way or another, are based on the probability distribution of the investment process.

Value at Risk (*VaR*) is at present the most popular downside risk measure.

It is defined as follows: *VaR* designates the loss of market value by a financial object such that the probability of equaling or exceeding this value over a set time frame is equal to the required level of confidence. The definition can be formally represented as follows (Jajuga [2000]):

$$P\,(Wt \le Wo - VaR) = \alpha, \tag{13}$$

where *Wo* – is the present value of the financial object (OF);
Wt – is the value of OF at the end of the time period under examination;
α – is the confidence level.

Making the following substitution in equation (13):

$$W\alpha = Wo - VaR$$

we arrive at:

$$P(Wt \le W\alpha) = \alpha\,. \tag{14}$$

*W*α is therefore an α-quantile, which means that *VaR* is a function of the financial object's price quantile.

The necessary and sufficient condition for determining the *VaR* is to have identified the distribution of the random variable *Wt*. In practice, estimations of this distribution are based on historical data and on assumptions on the mechanisms governing the variable.

In addition, we will now assume that we have some expert knowledge on the financial object under consideration. Let *Xt* stand for the fuzzy stochastic process which defines the value of the financial object at a point in time *t*.

We will adopt the notation *VaRq* to indicate the difference *Xt – Xo*.

VaRq then denotes the loss of market value by the financial object over the time period $<0,t>$.

The process *Xt* will be built by applying the procedures introduced in subchapter 2.

Having identified the distribution of the *Xt* process from the condition:

$$P(VaRq \text{ is } Aj) = \alpha \tag{15}$$

we can determine the fuzzy state *Aj*

where α represents the pre-defined confidence level.

Since condition (15) corresponds to condition (14), hence – by analogy – *VaRq* will be termed as *quasi VaR*.

Conclusion

The methodology of processing objective knowledge and expert knowledge for application in modeling investment processes presented in the paper creates new opportunities for efficient investment risk management. In particular, it can be applied to those risk management models that rely on the probability distribution of the investment process.

Finally, it should be observed that the efforts made to build a knowledge base from historical data and expert opinions are in unison with the ideas put forward in the June 2006 Basel Committee [on Banking Supervision] document entitled "Sound credit risk assessment and valuation for loans". One of the recommendations laid down in the document stresses to the importance of expert knowledge in modeling the parameters of credit risk.

[5] This particular model has been used for the forecasts of Euro rates (Walaszek Babiszewska [2005]).

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